Instability of FD-BPM when applied in Semiconductor Laser Modelling

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***Abstract*—We perform a stability analysis of the Crank-Nicolson based finite difference beam propagation algorithm for a medium with a complex refractive index distribution. We use typical waveguide parameters that correspond to an InGaAs-AlGaAs epitaxy. The results show that the FD-BPM scheme is unstable when performing the back propagation.**

I. Introduction

The beam propagation method (BPM) is used frequently in the design and modelling of semiconductor lasers (SL) [1]. When compared with SL models, which rely on the approximation that the laser operates in a single transverse mode, the BPM models provide insight into the intra-cavity laser mode evolution. The time domain models although superior in many respects, are significantly less numerically efficient than the BPM based models.

BPM based SL models are particularly well suited for the design and analysis of the high power laser diodes. These models were used for the design of broad area lasers, tapered lasers, tapered lasers with grating mirrors, external cavity tapered lasers, tapered lasers with patterned contacts, etc. [2, 3, 4, 5].

Initially fast Fourier transform based BPM (FFT-BPM) algorithms were used in the laser models [1]. However, more recently finite difference BPM (FD-BPM) algorithms have replaced the FFT-BPM ones [3, 4, 5]. This is because FD-BPM algorithms allow for an efficient implementation of the wide angle schemes and are fairly straightforward to implement in a programming language.

In this paper we study the stability of the FD-BPM in the context of the semiconductor laser modelling.

II. Theory

BPM algorithms are based on the approximation of the wave equation by the one way wave equation, c.f. [6]:

$$\frac{d\phi}{dz} = j\beta_r\phi - j\beta_r\left(\sqrt{\frac{\frac{\partial^2}{\partial x^2}+k^2-\beta_r^2}{\beta_r^2}+1}\right)\phi \qquad (1)$$

In (1) β_r is the reference propagation constant, k is the wavenumber, ϕ is the scalar potential while we assumed that the time dependence is: exp(jωt). The square root operator in (1) can be approximated using the Taylor expansion, which results in the well-known paraxial scheme [6]:

$$\frac{d\phi}{dz} = -j\frac{\beta_r}{2}\frac{\left(\frac{\partial^2}{\partial x^2}+k^2-\beta_r^2\right)}{\beta_r^2}\phi \qquad (2)$$

More accurate approximation is obtained when a Padé expansion is applied. In the case of the Padé approximation in the sum form (1) yields [6]:

$$\frac{d\phi}{dz} = -j\beta_r\sum_{i=1}^{n}\frac{a_{i,n}\frac{\left(\frac{\partial^2}{\partial x^2}+k^2-\beta_r^2\right)}{\beta_r^2}}{1+b_{i,n}\frac{\left(\frac{\partial^2}{\partial x^2}+k^2-\beta_r^2\right)}{\beta_r^2}}\phi \qquad (3)$$

One can observe that both equations (2) and (3) can be written in a compact, generic form:

$$\frac{d\phi}{dz} = M\phi \qquad (4)$$

Numerical solution of (4) applying the Crank-Nicolson (C-N) scheme yields [6]:

$$\phi(z+\Delta z) = \frac{1+\frac{\Delta z}{2}M}{1-\frac{\Delta z}{2}M}\phi(z) = P\phi(z) \qquad (5)$$

Introduction of finite difference approximations into (5) yields C-N version of the FD-BPM, which is one of the most often used FD-BPM algorithms. From (5) it follows that if all eigenvalues of (5) are within a unit circle the algorithm is stable. This is the case when the refractive index distribution is real. However, when the refractive index distribution has a nonzero imaginary part the stability of (5) is not obvious. In the next section we therefore study the properties of (5) for a typical semiconductor medium with gain.

III. Results

For a homogenous medium the properties of (5) can be studied using the Gershgorin circle theorem. However, the refractive index distribution in a semiconductor laser is not homogenous. We therefore calculate the eigenvalues for a typical waveguiding laser structure and then obtain the eigenvalues of the operator P. For this purpose we introduce a standard 3 point finite difference approximation for the second derivative in M and calculate all the eigenvalues of M. Once the eigenvalues of M are known the eigenvalues of P can be obtained from the mapping (5). We consider a typical core-clad waveguiding structure that is obtained upon applying the effective index method, c.f. [2]. We take from [2] the modelling parameters that correspond to an InGaAs-AlGaAs semiconductor laser operating at 980 nm. The core and cladding effective index is respectively 3.31 and 3.008 while the waveguide width is 4 μm. The confinement factor equals 0.02.

Figure 1 shows the dependence of the magnitude of the eigenvalues of P on the mode number for the paraxial and wide angle schemes. The modes were sorted in the ascending order so that the mode with the largest number corresponds to the mode with the largest value of the propagation constant (which is the fundamental waveguide mode). In this simulations Δz = 0.1 μm, Δx = 1 μm while the reference propagation constant equals to the propagation constant of the fundamental mode, which was calculated consistently using the finite difference method with the assumed value of the transverse mesh size. The size of the numerical window equals 40 μm. These results show that all eigenvalues of P are within the unit circle and the algorithm is hence stable. However, if Δz = - 0.1 μm the situation changes (Fig.2). In this case only the fundamental mode propagation constant is mapped within the unit circle while the propagation constants of the other modes are mapped outside of the unit circle. Hence, the algorithm is not stable.

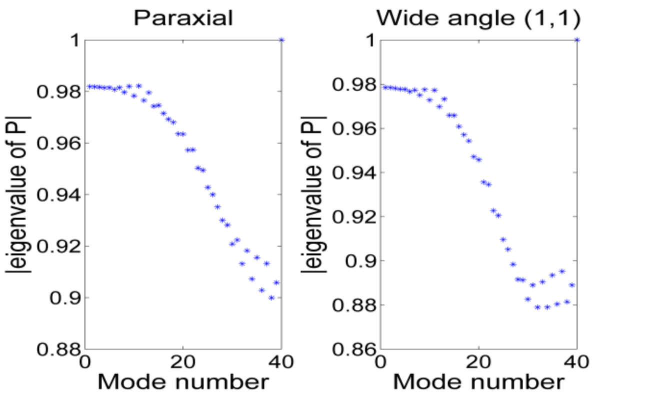

Fig. 1. Dependence of the magnitude of the eigenvalues of P on the mode number for the paraxial and wide angle scheme, Δx = 1 μm while Δz = 0.1 μm.

As a result of the algorithm instability the field cannot be back propagated using the C-N FD-BPM algorithm. This last point is illustrated in Figure 3, which shows an attempt to back propagate the fundamental mode in the studied waveguide semiconductor laser example using paraxial C-N FD-BPM within a 2D model [2]. Despite the fact that the structure is longitudinally invariant, higher order modes are excited by numerical round off errors and they dominate gradually.

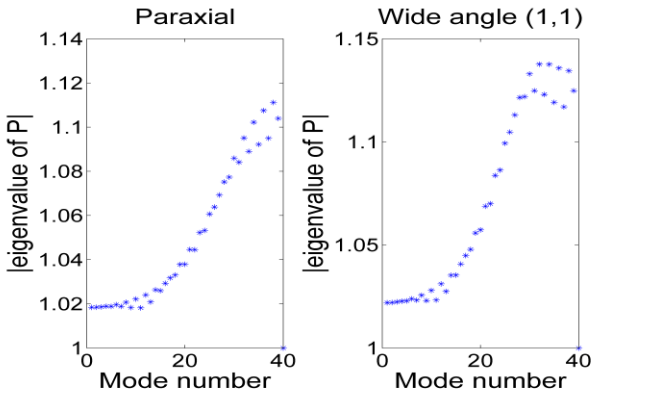

Fig. 2. Dependence of the magnitude of the eigenvalues of P on the mode number for the paraxial and wide angle scheme, Δx = 1 μm while Δz = - 0.1 μm.

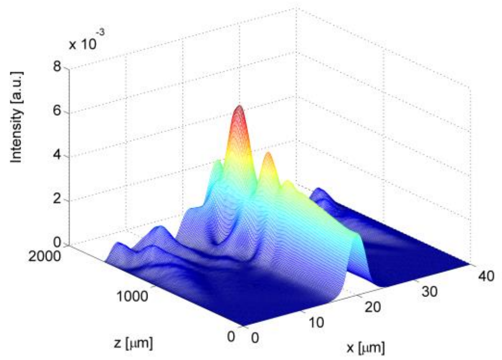

Fig. 3. Intensity distribution calculated by backpropagating the fundamental mode using C-N FD-BPM within 1600 μm long waveguide, Δx = 0.1 μm while Δz = - 0.1 μm.

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