Simulation of Kerr-nonlinear waveguide structures by an eigenmode expansion method

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***Abstract*—We present a new implementation of the eigenmode expansion technique for modeling Kerr-nonlinear waveguide structures. The formulation uses numerically stable scattering matrices and a perturbation approach based on the rigorous coupled-mode theory.**

Keywords—eigenmode expansion and propagation; optical waveguides; coupled-mode theory; Kerr-nonlinearity

I. Introduction

Rigorous electromagnetic simulation is of fundamental importance in the analysis and design of new photonic devices. Among various computational techniques, *linear* waveguide structures can be effectively simulated by using the eigenmode expansion technique (EME) [1,2]. The approach is particularly advantageous for the structures composed of longitudinally uniform waveguides ("sections"). In principle, EME can deal with the structures of arbitrary length and readily provides device characteristics such as transmission, reflection or radiation loss. An extension of EME for *Kerr-nonlinear* structures has been already demonstrated [3]. The nonlinear technique uses spatial discretisation of nonlinear sections and an iterative procedure that requires a repeated calculation of eigenmodes. The aim of this work is to present an alternative technique [4], labeled as NL-EME, which solves the modal propagation in the nonlinear sections by using a perturbation approach based on the rigorous coupled-mode theory. In this way, the recalculation of eigenmodes is avoided and thus one of the main advantages of EME maintained.

II. Formulation

A nonlinear structure, such as in Fig. 1, is described with the dielectric function $\varepsilon = \varepsilon_0 + \Delta\varepsilon \equiv \varepsilon_0 + \gamma|\vec{E}|^2$, where $\varepsilon_0(x,y,z)$ is the linear dielectric function, $\gamma(x,y,z)$ is the constant related to the Kerr-nonlinear response and $\vec{E}(x,y,z)$ is the electric field. The structure is divided into the longitudinally uniform sections, i.e., inside of any section, ε_0 and γ are functions of (x,y) only and the field is expanded in terms of the linear eigenmodes as [1,2]

$$\vec{E}_\perp = \sum_m [f_m \exp(-i\beta_m z) + b_m \exp(i\beta_m z)]\vec{e}_{m\perp}, \quad (1)$$

$$\vec{H}_\perp = \sum_m [f_m \exp(-i\beta_m z) - b_m \exp(i\beta_m z)]\vec{h}_{m\perp}. \quad (2)$$

Here, $\vec{H}(x,y,z)$ is the magnetic field, $\perp$ denotes the transverse field components, β_m is the propagation constant of the m-th eigenmode, $\vec{e}_m(x,y)$ and $\vec{h}_m(x,y)$ are the corresponding electric and magnetic modal field profiles, respectively, and $f_m(z)$ and $b_m(z)$ are the amplitudes of the forward and backward propagating eigenmodes, respectively. The eigenmodes are normalized as in [4].

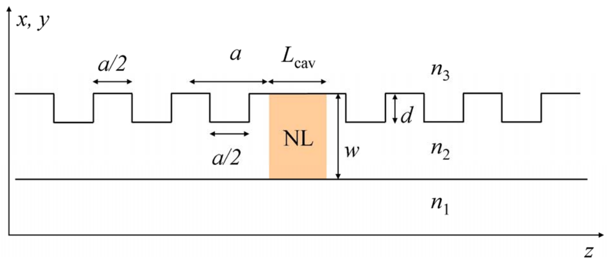

Fig. 1. Nonlinear waveguide cavity (marked with NL) with the distributed Bragg grating reflectors (DBR). The waveguide width is $w = 0.5\,\mu$m. The cavity has the length L_{cav} and is surrounded by the two identical DBR sections with the period $a = 0.45\,\mu$m and depth of the teeth $d = 0.2\,\mu$m. The refractive indices of the substrate, the waveguide core, and the superstrate are $n_1 = 1.44$, $n_2 = 1.98$ and $n_3 = 1$, respectively. Each DBR section contains 32 periods and provides Bragg reflection at the operating wavelength $\lambda = 1.55\,\mu$m.

For linear structures ($\gamma = 0$), the technique follows the formulation which employs the scattering matrices (S-matrices) [2]; the formalism is essential to prevent numerical instabilities that are related to propagation of evanescent modes.

For nonlinear structures, we consider $\Delta\varepsilon \neq 0$ as a small perturbation of ε_0 and use the rigorous coupled-mode theory. The procedure leads to the following nonlinear system of coupled differential equations for the amplitudes f_m and b_m:

$$\begin{aligned}\frac{df_m}{dz} = &-i\sum_n K_{mn}^{(-)} f_n \exp[i(\beta_m - \beta_n)z] \\ &-i\sum_n K_{mn}^{(+)} b_n \exp[i(\beta_m + \beta_n)z],\end{aligned} \quad (3)$$

This work was supported by Ministry of Education, Youth, and Sports of the Czech Republic under contract LD14008.

$$\frac{db_m}{dz} = i\sum_n K_{mn}^{(+)} f_n \exp[i(-\beta_m - \beta_n)z] + i\sum_n K_{mn}^{(-)} b_n \exp[i(-\beta_m + \beta_n)z], \tag{4}$$

where the coupling coefficients $K_{mn}^{(\pm)}$ are given by

$$K_{mn}^{(\pm)} = \iint \left(\Delta\varepsilon \vec{e}_{m\perp} \cdot \vec{e}_{n\perp} \pm \frac{\varepsilon_0 \Delta\varepsilon}{\varepsilon_0 + \Delta\varepsilon} e_{mz} e_{nz} \right) dxdy. \tag{5}$$

Solution of the coupled system (1)-(5) is straightforward for the structures in which propagation of backward modes can be neglected; this simplification was used in [5] for investigation of nonlinear plasmonic coupler. However, in the general case, it is necessary to use an iterative technique developed in [4]. In particular, at each iteration step, we obtain linear problem, which can be efficiently solved by using S-matrices; in this way, the nonlinear algorithm naturally extends the linear technique.

III. NUMERICAL EXAMPLE

To demonstrate the technique we present a simulation of a nonlinear waveguide cavity with the distributed Bragg grating reflectors (DBR), see Fig. 1. The structure is excited with the fundamental TE mode with the amplitude of the electric field E_m. The nonlinearity strength is characterized with a parameter $\eta = \gamma |E_\mathrm{m}|^2 / (2n_2)$. The calculation was performed with 20 eigenmodes (in each section) and the discretisation step $\Delta z = 0.01\,\mu\mathrm{m}$.

Fig. 2(a) shows the modal transmissivity as a function of the cavity length L_cav for the various levels of the nonlinearity. As expected, the nonlinearity shifts the peak to the lower values of L_cav and a narrow bistable region is observed. The bistability is also demonstrated in Fig. 2(b), which presents a nonlinear characteristic of the device. Clearly, the iterative procedure used in NL-EME can describe the bistable response and converges even in the case of discontinuities.

The example also demonstrates a very attractive feature of the technique. Often, it is necessary to simulate complex structures with short nonlinear sections. In this case, S-matrices for linear sections (e.g., S-matrix describing whole DBR for the structure in Fig. 1) are calculated once and only the S-matrices describing the nonlinear sections must be iteratively recalculated.

IV. CONCLUSION

We presented a new implementation of the eigenmode expansion technique for modeling Kerr-nonlinear waveguide structures. The technique combines perturbation approach based on the rigorous coupled-mode theory and numerically stable scattering matrices. The paper will introduce the model and theoretical formulation of NL-EME. Then we will present numerical results for typical structures that can be simulated and compare NL-EME with other established techniques.

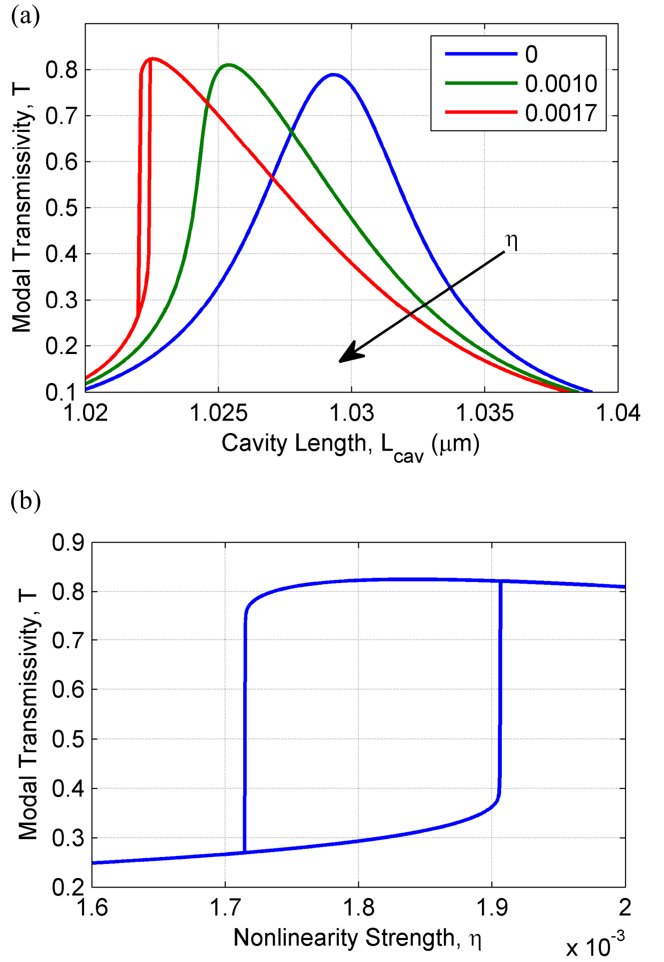

Fig. 2. (a) Modal transmissivity T vs. the cavity length L_cav for various values of the nonlinearity strength η shown in the box. (b) Modal transmissivity T vs. the nonlinearity strength η for the cavity length $L_\mathrm{cav} = 1.022\,\mu\mathrm{m}$. The other structure parameters are as in Fig 1.

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