Smart Techniques for Modelling Nanophotonic Circuits

(Invited)
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***Abstract*— Efficient computational methods for modeling nanophotonic devices are presented. These methods include efficient sensitivity analysis based on full wave electromagnetic solvers. It also includes analytical modeling for plasmonic devices using impudence approach. These methods proved to be effective and easy to implement.**

I. Introduction

Modelling of photonics and nanophotonics using full wave electromagnetic simulation is highly demanding. This is mainly due to the fact that photonic devices are in general electrically long. Nanophotonics such as plasmonic devices may have more compact size and this may suggest less computational resources. However, due to the subwavelength nature of these devices, the grid size has to be much smaller than the feature size in order to sustain acceptable accuracy. This in turn increases the computational resources for such devices. Hence, efficient modelling techniques of these devices is of prime importance.

In addition, sensitivity analysis of these devices are essential for tolerance and yield analysis. The sensitivity information is also important for design optimization as it allows for using gradient based optimization approach. Obtaining the sensitivity information from electromagnetic solver is however, highly costly. This is mainly because these solvers do not provide such information and require many additional simulations to obtain sensitivity of all the design parameters using the traditional perturbation approach based on finite difference method. For example, for N design parameters, addition 2N simulations are required to obtain the sensitivity of all these parameters using central finite difference (CFD) method.

In this work, we discuss two novel approaches for efficient modelling of photonic and nanophotonic devices. The first one is based on efficient extraction of the sensitivity information based on full electromagnetic solver using adjoint variable method. The second approach, is based on providing a completely analytical modelling approach for the plasmonic devices using simple impedance model.

II. Sensitivity Analysis Using Adjoint Variable Method

This approach is based on creating an adjoint system that allows for estimating the sensitivity information directly from the same electromagnetic simulation. This approach highly depends on the system of equation for the electromagnetic solver. Hence, it varies by varying the numerical method. This approach allows for estimating the response and the sensitivity of the response to various design parameters from the same simulation. This response can be the scattering parameters, the transmission, the energy and it can be also the modal parameters.

This approach has been applied for finite difference time domain (FDTD)[1]-[3] and beam propagation method BPM[4],[5]. This approach has been also utilized in design optimization of novel devices such as wide band optical switch [6] and polarization splitter [7].

Efficient sensitivity of quantum structures has been also recently proposed using the adjoint technique. This approach utilizes the time independent Schrodinger approach to get the sensitivity of the energy level and wavefunction of any quantum structure[8]. It has been also applied to finite element method.

In general, the electromagnetic solver can be casted in the following form

$$\boldsymbol{A}(p)\boldsymbol{x} = \boldsymbol{b} \tag{1}$$

where $\boldsymbol{A}$ and $\boldsymbol{b}$ are the system matrix and the excitation vector, respectively. In (1), $\boldsymbol{x}$ is the state variable which can be the electric field. By differentiating (1) with respect to the nth design parameter p_n and rearranging, we get

$$\frac{\partial \boldsymbol{x}}{\partial p_n} = \boldsymbol{A}^{-1}\left[\frac{\partial b}{\partial p_n} - \frac{\partial \boldsymbol{A}}{\partial p_n}\boldsymbol{x}\right] \tag{2}$$

The sensitivity of the objective function f can be calculated using the chain rule as follow

$$\frac{\partial f}{\partial p_n} = \frac{\partial^e f}{\partial p_n} + \frac{\partial f}{\partial \boldsymbol{x}} \bullet \frac{\partial \boldsymbol{x}}{\partial p_n} \tag{3}$$

where $\partial^e / \partial p_n$ represents the explicit dependence of f on the design parameter p_n. By substituting (2) into (3), the sensitivity of the objective function can be obtained as follows

$$\frac{\partial f}{\partial p_n} = \frac{\partial^e f}{\partial p_n} + \frac{\partial f}{\partial \boldsymbol{x}}\boldsymbol{A}^{-1}\left[\frac{\partial b}{\partial p_n} - \frac{\partial \boldsymbol{A}}{\partial p_n}\boldsymbol{x}\right] \tag{4}$$

Now, we define the adjoint variable $\hat{\boldsymbol{x}}$ as

$$\hat{\boldsymbol{x}}^T = \frac{\partial f}{\partial \boldsymbol{x}}\boldsymbol{A}^{-1} \tag{5}$$

This adjoint variable can be obtained by solving the system of equation:

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$$A^T \hat{x} = \left(\frac{\partial f}{\partial x}\right)^T \quad (6)$$

The system of equation given in (6) is called the adjoint system. Now by substituting (5) into (4), the sensitivity of the objective function is given by:

$$\frac{\partial f}{\partial p_n} = \frac{\partial^e f}{\partial p_n} + \hat{x}^T \left[\frac{\partial b}{\partial p_n} - \frac{\partial A}{\partial p_n} x\right] \quad (7)$$

Since the LU factorization of the matrix A is readily available from solving the original system given in (1), the adjoint variable $\hat{x}$ can be obtained efficiently using (6) by forward–backward substitution. The derivative of the system matrices can be easily obtained using perturbation approach without solving the perturbed system.

$$\frac{\partial A_i}{\partial p_j} \approx \frac{\left(A_i(p_j + \Delta p_j) - A_i(p_j)\right)}{\Delta p_j} \quad (8)$$

Thus, the sensitivity expression in (7) can be solved efficiently.

III. Analytical Approach for plasmonic waveguides

Plasmonic waveguides attract the attention in the last decade due to its unique ability to guide the light in subwavelength scale. This unique feature suggests various applications including; optical interconnects and on chip sensing. The plasmonic slot waveguide (PSW) is considered as the most suitable configuration for the aforementioned applications. This waveguide consists of a dielectric slot surrounded by a Nobel metal that exhibits surface plasmon polariton (SPP) resonance at the operating wavelength. PSW also enjoys the unique ability to transmit the light through sharp bends with negligible loss. This feature paves the way for subwavelength functional device size.

Modelling of the plasmonic devices is highly demanding as it requires very fine mesh to model the subwavelength features with good resolution. 3D FDTD modelling of such structures is highly demanding both in time and memory resources. Efficient modelling of the plasmonic devices is essential to allow for fast design optimization.

Due to the unique ability of the PSW to couple light efficiently to orthogonal directions and through sharp bends. Various junctions such as X and T junction can be easily modelled using impedance model to estimate the power coupled to each arm in the junction. This model utilizes the waveguide impedance to estimate the total loading impedance and hence distribute the power accordingly.

The reflections and transmission at each port can be easily estimated using transmission line theory. The reflection and transmitted field can be easily obtained using

$$r_m = \left|\frac{Z_t - Z_m}{Z_t + Z_m}\right| . e^{-2\gamma l_m} \quad \text{, and} \quad (9)$$

$$t = \left|\sqrt{\frac{Z_n}{Z_t}} \times \frac{2\sqrt{Z_m Z_t}}{(Z_m + Z_t)}\right| . e^{-\gamma(l)} \quad (10)$$

where

$$Z_m(\omega, d) = \frac{\beta_{PSW}(\omega, d)\, d}{\omega \varepsilon(\omega)} \quad (11)$$

where Z_m and Z_n are the waveguide impedance of the input port section and the loading sections, l is the length of the waveguide section, and d and the width of the slot.

Using this simple model, the reflection and transmission of each junction can be estimated efficiently [10]. Scattering matrix approach can be utilized to estimate the final response analytically.

The approach has been recently utilized to obtain the analytical expression for various plasmonic devices including; fnano resonator [10], in line filter [11], plasmonic mesh structure [12] and plasmonic power splitter [13].

This approach provides accuracy comparable to the FDTD and alleviates the need for electromagnetic simulation. It also allows for physical insight of the device performance.

IV. Conclusion

Various efficient approaches for modelling photonics and nanophotonic devices are presented and discussed. These approaches open the door for fast and efficient optimization for wide range of applications

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