Theoretical analysis of passively mode-locked inhomogeneously broadened lasers

Alexander Pimenov
Weierstrass Institute
Berlin, Germany 10117
Email: pimenov@wias-berlin.de

Andrei G. Vladimirov
Weiersrrass Institute
Berlin, Germany 10117

***Abstract*—We consider a quasi-two-level travelling wave model of an inhomogeneously broanened laser. We propose a new numerical method to solve these equations, and perform numerical simulations to study the effect of inhomogeneous broadening on the properties of mode-locking regime.**

I. Introduction

Passively mode-locked lasers are used for generation of short optical pulses that find applications in the areas such as high speed communications and medical diagnostics. In particular, quantum dot (QD) and quantum dash mode locked semiconductor lasers possess characteristics that are promising for the use in the optical communications [1]. Such lasers exhibit high inhomogeneous broadening of the gain medium due to inhomogeneity of the ensemble of quantum dots in respect to their size, shape and composition, which contributes significantly to the pulse shaping process. In quantum-dot lasers under the bias conditions the inhomogeneous broadening width at half-maximum (from 21 meV to 50 meV) is larger than homogeneous broadening width (19 meV) [2]. In this work, we consider Maxwell-Bloch equations for the elecrical field envelopes of the waves travelling backward and forward coupled to the quasi-two-level equations for polarizations and carrier densities of the quantum dot groups emitting at different central frequencies through the integral over these frequencies [3]. Similar model was previously used to describe inhomogeneous broadening of gas lasers, where the authors simplified the system by either introducing auxiliary macroscopic momentum variables [4], or discretizing the integral [3], [5], [6]. We integrate the partial integro-differential equations numerically with the help of an efficient spectral method based on Hermite functions as the basis functions to study the effect of the inhomogeneous broadening width on the properties of mode-locked regime.

II. Model and the spectral method

A. Quasi-two-level travelling wave model

We consider a travelling wave model for a Fabry-Perot inhomogeneously broadened laser in non-dimensional form, which is obtained from quasi-two-level Maxwell-Bloch equations under standard mean-field, effective-index, and slowly varying envelope approximations [3]

$$\frac{\partial E^{\pm}}{\partial t} \pm \frac{\partial E^{\pm}}{\partial z} = -\frac{\beta}{2}E^{\pm} + \int_{-\infty}^{\infty} P^{\pm} f(\omega) d\omega, \tag{1}$$

$$\frac{\partial P^{\pm}}{\partial t} = (-\Gamma + i\omega)P^{\pm} + \frac{g}{2}NE^{\pm}, \tag{2}$$

$$\frac{\partial N}{\partial t} = j_0 - \gamma N + \Re(P^{+}E^{+*} + P^{-}E^{-*}), \tag{3}$$

where $E^{\pm}(t,z)$ are envelopes of the electric field travelling forward and backward, $P^{\pm}(\omega,t,z)$ represents two-level electric polarization, $N(\omega,t,z)$ represents population difference, β describes linear internal losses in the intracavity medium, g is the differential gain/loss parameter, Γ is polarization decay rate, γ is population difference relaxation rate, j_0 describes linear gain/absorption. Boundary conditions are given by

$$E^{+}(0,t) = \sqrt{\kappa_1}E^{-}(0,t), \qquad E^{-}(l,t) = \sqrt{\kappa_2}E^{+}(l,t), \tag{4}$$

with the reflectivities on the left and right facets $\kappa_{1,2}$. The normalized spectral distribution $f(\omega)$ most commonly takes the form of the Gaussian distribution

$$f(\omega) = \frac{1}{\sigma_D\sqrt{2\pi}} \exp\left(-\frac{(\omega-\omega_0)^2}{2\sigma_D^2}\right), \tag{5}$$

where σ_D is the width of inhomogeneous broadening at half-maximum, ω_0 is the detuning between the atomic line center and frequency of the cavity mode.

B. Hermite functions

Next, we propose the numerical method to solve (1)-(3). For that we will project the variables $P^{\pm}, N$ as the functions of ω on the finite subset of the basis formed by Hermite functions

$$\psi_m(\omega) = (m!2^m\sqrt{\pi})^{-1/2} e^{-\omega^2/2} H_m(\omega). \tag{6}$$

Function $H_m(\omega)$ is a Hermite polynomial, which is defined by the recursive formula

$$\begin{gathered} H_0(\omega) = 1, \qquad H_1(\omega) = 2\omega, \\ H_{m+1}(\omega) = 2\omega H_m(\omega) - 2mH_{m-1}(\omega). \end{gathered} \tag{7}$$

C. Finite-dimensional 1D+1 problem

We multiply (2)-(3) by ψ_0, make a change of variables $\psi_0 P^{\pm}, \psi_0 N \rightarrow P^{\pm}, N$ and a coordinate change $(\omega - \omega_0)/(\sqrt{2}\sigma_D) \rightarrow \omega$, and make the projection of the variables $P^{\pm}, N$ and of the equations (2)-(3) on the subset of Hermite

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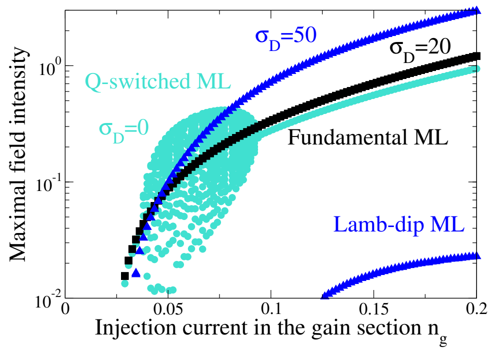

Fig. 1. Bifurcation diagram obtained by changing the parameter n_g for $\sigma_D = 0, 20, 50$. Here $\Gamma_g = \Gamma_q = 40, \gamma_g = 0.01, \gamma_q = 1, n_q = -1.6$.

functions $\psi_0, ..., \psi_M$ to obtain the following system of partial differential equations

$$\frac{\partial E^{\pm}}{\partial t} \pm \frac{\partial E^{\pm}}{\partial z} = P_0^{\pm}, \tag{8}$$

$$\frac{\partial P_m^{\pm}}{\partial t} = (-\Gamma + i\omega_0)P_m^{\pm} + \frac{g}{2}N_m E^{\pm} + i\sigma_D(\sqrt{m}P_{m-1}^{\pm} + \sqrt{m+1}P_{m+1}^{\pm}), \tag{9}$$

$$\frac{\partial N_m}{\partial \tau} = n_{0m} - \gamma N_m - \Re(E^+ P_m^{+*} + E^- P_m^{-*}), \tag{10}$$

$$E^{\pm}(0,z) = E_0^{\pm},\ P_m^{\pm}(0,z) = P_{0m}^{\pm},\ N_m(0,z) = N_{0m}, \tag{11}$$

where $P_m^{\pm}, N_m$ are the moments of polarization and population difference, $P_{M+1}^{\pm} \equiv P_{-1}^{\pm} \equiv 0$, and due to orthonormality of Hermite functions $n_{0m} = j_0\delta_{0m}$ for all $m = 0..M$, where $\delta_{ij} \equiv 0$ for $i \neq j$ is a Kronecker-delta function.

III. Numerical Results

We solve equations (8)-(10) for the two-section semiconductor laser with a gain and an absorber section using the discretization scheme similar to the one reported in [7], [8]. Since the number of moments reaches up to $M = 200$ in our simulations, we realize this scheme in parallel.

We see on Fig. 1 that without inhomogeneous broadening ($\sigma_D = 0$) with the increase of injection current first the Q-switching regime is stable and then fundamental ML regime gains stability. With the increase of the width of inhomogeneous broadening ($\sigma_D = 20$, whereas $\Gamma_g = \Gamma_q = 40$) we observe that Q-switching regime is supressed and ML regime is stable for the considered range of injection current. Moreover, by further increasing $\sigma_D = 50$ we see that the second peak appears in the pulse profile. By examining the spectral profile of the pulses for high σ_D (see Fig. 2,3) we see that the appearance of the small peaks near the pulse corresponds to the formation of the Lamb dip in the spectral profile.

IV. Conclusion

We have considered travelling wave model of an inhomogeneously broadened laser and proposed and realized efficient numerical method for the parallel simulation. We have shown that inhomogeneous broadening suppresses Q-switching and is responsible for the formation of the Lamb dip in the spectral profile of the pulse.

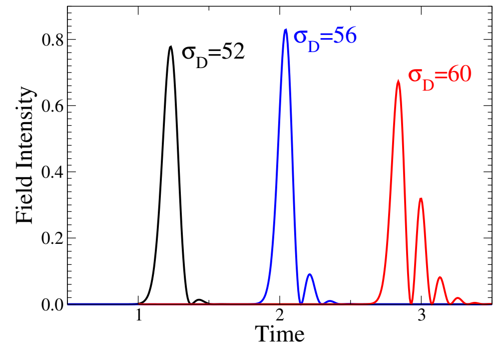

Fig. 2. Pulse profiles for $\sigma_D = 52, 56, 60$.

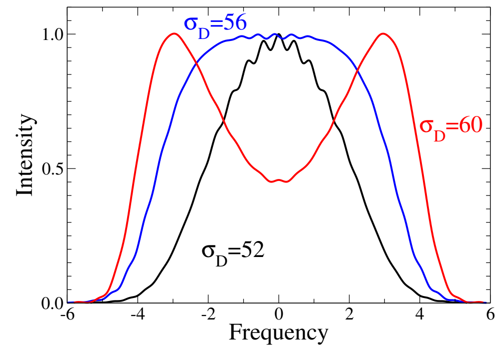

Fig. 3. Spectral profiles of the pulses for $\sigma_D = 52, 56, 60$.

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