Modeling of multimode laser dynamics by means of delay differential equations

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***Abstract*—We discuss an approach to the analysis of nonlinear dynamics in multimode semiconductor lasers based on the use of delay differential equations (DDEs) for the electric field envelope and carrier density in nonlinear intracavity media. We consider DDE models of a mode-locked semiconductor laser generating short optical pulses and a multistripe laser array with external feedback. We present the results of numerical simulations of different dynamical regimes in these lasers and discuss asymptotic approaches for the stability analysis of different operation regimes.**

I. Introduction

Conventional methods to study the dynamics of multimode semiconductor lasers are based either on the numerical simulations of partial differential traveling wave equations or on the analysis of large systems of coupled equations for the amplitudes of individual modes. Here we discuss an alternative approach to the analysis of the dynamics of multimode lasers based on the use of delay differential equations (DDEs). We show that this approach allows not only to perform a comprehensive numerical bifurcation analysis of different laser operation regimes, but also to obtain some analytical results.

Here we introduce two DDE models of multimode semiconductor lasers. The first model describes the generation of short optical pulses in a passively mode-locked laser. This model proposed in [1]–[3] can be considered as an extension of the Haus master equation to the case of semiconductor lasers with large gain and loss per cavity round trip. Later this model was generalized to study mode-locking in quantum dot lasers [4]. Furthermore, a modification of the DDE mode-locking model was very recently adopted to describe a Fourier mode-locking regime in a ring laser with tunable optical bandpass filter [5].

The second model describes a multistripe laser array with an external off-axis feedback, which was studied experimentally in [6]. This model is based on a set of DDEs for the electric field envelope, homogeneous component of the carrier density, and the transverse carrier grating. Using the DDE model of the multistripe laser array we study different dynamical instabilities of the operation regime corresponding to anti-phase synchronization of the neighboring stripes in the array.

II. DDEs for a Passively Mode-Locked Semiconductor Laser

A DDE model of a passively mode-locked semiconductor laser was derived in [1]–[3] using the so-called lumped element method and assuming unidirectional lasing in a ring cavity with Lorentzian shape of the spectral filtering. This model governing the time evolution of the electric field envelope A at the entrance of the laser absorber section, saturable gain G, and saturable loss Q, reads:

$$\gamma^{-1}\partial_t A + A = \sqrt{\kappa}e^{(1-i\alpha_g)G/2-(1-i\alpha_q)Q/2-i\varphi}A_T, \quad (1)$$

$$\partial_t G = g_0 - \gamma_g G - e^{-Q}\left(e^G-1\right)|A_T|^2, \quad (2)$$

$$\partial_t Q = q_0 - \gamma_q Q - s\left(1-e^{-Q}\right)|A_T|^2. \quad (3)$$

Here $A_T = A(t-T)$, T is the cavity round trip time, γ is the spectral filtering bandwidth, s is the effective saturation parameter, and κ is the linear attenuation factor per cavity round trip. The parameters g_0 (q_0), γ_g (γ_q), and α_g (α_q) describe linear gain (absorption), carrier relaxation rate, and linewidth enchancement factor in the amplifying (absorbing) section. The DDE model (1) - (3) can be considered as an extension of the classical Haus model to the case of large gain and loss per cavity round trip. A modification of the DDE model was recently successfully applied to describe the characteristic features of a Fourier domain mode-locked laser observed experimentally [5].

III. DDE Model of a Multistripe Laser Array

Schematic representation of a multistripe laser array with external off-axis feedback is shown in Fig. 1. Here, κ_1 and κ_2 describe the reflectivity of the left laser facet and feedback mirror, respectively, α is the angle of the tilt of the feedback mirror, and L is the distance from the right laser facet to this mirror. The distance L is assumed to be much larger than the width w and the length l of the array, $L \gg w, l$. The time required for the light to travel from the array to the feedback mirror and back is given by $\tau = 2L/c_0$, where c_0 is the velocity of light in vacuum.

When the tilt angle α of the external mirror is properly adjusted, so that the adjacent stripes are syncronized anti-phase, the array emits a double lobed far field pattern with a pronounced output lobe at the angle α and a slightly suppressed feedback lobe at the opposite angle $-\alpha$. This behavior was observed experimentally [6] and reproduced in numerical simulations using a 2+1 dimensional traveling wave model [7]. Using the approach similar to that discribed in [1]–[3] we have derived the following DDE model of the multistripe array shown in Fig. 1.

$$\Gamma^{-1}\partial_t A + A = (1-i\alpha_H)\kappa_1\kappa_2 e^{(1-i\alpha_H)G_T}H_T A_T, \quad (4)$$

$$\partial_t G = G_0 - \gamma G - |A|^2(e^G-1)(1+\kappa_1^2 e^G). \quad (5)$$

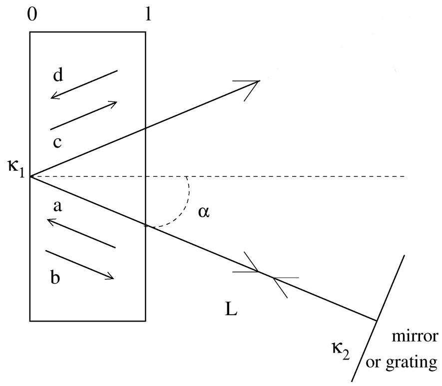

Fig. 1. Schematic view of a multistripe laser array with off-axis feedback.

$$\partial_t H = \quad H_0 - \gamma H - |A|^2 H\{\tfrac{1-i\alpha_H}{2} e^G[\kappa_1^2(2e^G - 1) + 1] + \tfrac{1+i\alpha_H}{2} \tfrac{e^G - 1}{G}(\kappa_1^2 e^G + 1)\}. \quad (6)$$

Here $A(t)$ is the electric field amplitude, $G(t)$ is the transversely homogeneous component of the saturable gain, and $H(t)$ is the amplitude of the transverse carrier grating, $H(t) = e^{i\phi(t)}|H(t)|$. The parameters H_0 and G_0 act as pump parameters for the homogeneous component of the carrier density G and the carrier grating H, respectively, and α_H is the linewidth enhancement factor. The subscript T denotes delayed argument, $\phi_T = \phi(t-T)$, $H_T = H(t-T)$, and $G_T = G(t-T)$. The delay time is $T = 2(L+l)/c_0$. In the derivation of (4)-(6) we have assumed that the array operates sufficiently close to the lasing threshold and/or the transverse grating in the active medium is sufficiently weak [8]. However, despite the above mentioned approximations, the results obtained with the model (4)-(6) are in a good qualitative agreement with those of numerical simulations with the 2+1 dimensional traveling wave model [7].

Numerical analysis of the model equations (4)-(6) has been performed using the routines for direct numerical integration of DDEs and the software package DDE-Biftool [9]. Figure 2 illustrates some results of this analysis. In this figure two branches of CW regimes, $CW1$ and $CW2$, correspond to two different longitudinal modes of the multistripe laser array with external feedback. With the increase of α_H the solution $CW1$ looses and the solution $CW2$ gains stability via subcritical Andronov-Hopf bifurcations giving rise to a branch of unstable periodic solutions. Bistability between the two CW solutions is observed within a certain range of the linewidth enhancement factors. At sufficiently large α_H the solution $CW2$ becomes unstable again via a supercritical Andronov-Hopf bifurcation leading to the appearance of stable periodic solution $P1$.

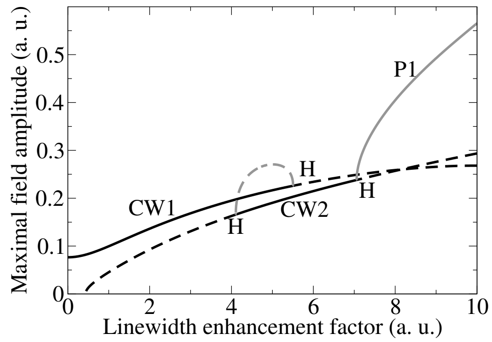

Fig. 2. Bifurcation diagram of Eqs. (4)-(6) obtained with help of DDE-Biftool package. Linewidth enhancement factor α_H is used as a bifurcation parameter. Solid black line: stable CW solution. Dashed black line: unstable CW solution. Solid gray line: stable periodic solution. Solid dashed line: unstable periodic solution. H indicates an Andronov-Hopf bifurcation point. Parameter values are: $T = 2.5$, $\gamma = 0.065$, $\Gamma = 2/T$, $\kappa_1 = 0.95$, $\kappa_2 = 0.9$, $G_0 = 0.07$.

IV. Conclusion

We have discussed an approach to analyze the dynamics of multimode semiconductor lasers based on the use of DDEs. In particular a DDE mode-locking model has been proven a useful tool for mathematical analysis of passively and hybrid mode-locked monolithic semiconductor lasers, as well as some other devices, such as frequency swept and Fourier domain mode-locked lasers [5].

We have shown that characteristic features of the dynamics of multistripe laser arrays with external off-axis feedback observed earlier in numerical simulations of the traveling wave model [7] are well reproduced with the help of the reduced DDE model. Bifurcation analysis of the DDE model indicates that at sufficiently large values of the injection current and the linewidth enhancement factor different instabilities of CW regimes can develop in the system. In particular, an Andronov-Hopf bifurcations are responsible for the destabilization of CW regimes and appearance of single- and multimode pulsations.

Acknowledgment

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