Evaluating the Purcell Factor in Hyperbolic Metamaterials

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***Abstract*—We compare two simulation methods of calculating the Purcell factor in a hyperbolic metamaterial and validate our simulations with measurements. We calculate the Purcell factor from a direct calculation of the local density of states from the band structure of the periodic unit cell. We compare this method with calculations of the Purcell factor obtained from the finite difference time domain (FDTD) calculations of the impedance of a dipole antenna located inside the structure. We show that we can study the Purcell factor in transverse electric (TE) and transverse magnetic (TM) polarisations by using electric and magnetic dipoles, we support our argument by analysing the dispersion relations of the structure.**

I. Introduction

The Purcell factor (F) is the ratio of rate of spontaneous emission in a medium to the rate of spontaneous emission in vacuum. Engineering the Purcell factor and increasing the rate of spontaneous emission has applications in the development of lasers, single photon emitters. Purcell factor is directly related to the local density of states (LDOS). Hyperbolic metamaterials [1] have gained interest in recent times due to their large LDOS to achieve broadband enhancement of the Purcell factor.

In the semi-classical approximation, the Purcell factor is the same as the ratio of change in radiated power of a classical dipole. Using this analogy, the Purcell factor for any structure can be obtained by measurements of the change in antenna impedance [2]. This method has gained attention recently [3] because it provides a flexible way to probe the Purcell factor and LDOS. The impedance method has the added advantage that by replacing the electric dipole by a magnetic dipole we can obtain the electric and magnetic Purcell factors separately.

We analyse the Purcell factor for a simple hyperbolic metamaterial structure [4] in the frequency range 5-15 GHz. We use FDTD calculations of impedance of electric and magnetic dipole antennas to calculate the Purcell factor. We compare the impedance method with the Purcell factor obtained from calculation of LDOS, which we obtain from the band structure of the periodic unit cell of the metamaterial. We obtain the same behaviour in the Purcell calculated from both methods. The Purcell factor is enhanced in the TM polarisation while it is suppressed in TE polarisation, we give an explanation of effect analysing dispersion relations of the structure.

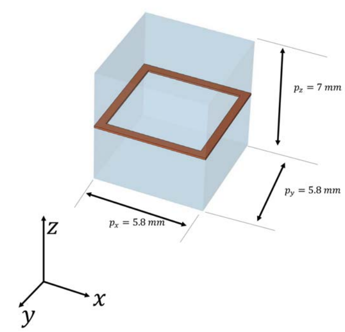

Fig. 1. Unit cell of the hyperbolic metamaterial structure, fabricated of 0.1 mm thick copper grids. The periodicity $p_x = p_y = 5.8$ mm, $p_z = 7.0$ mm in the x, y and z directions.

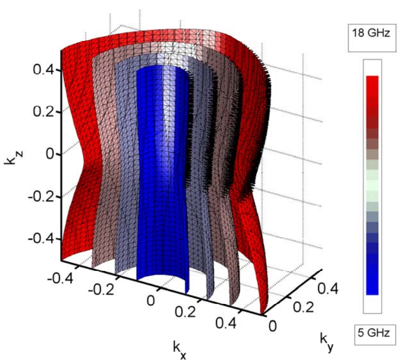

Fig. 2. Hyperbolic isofrequency surfaces obtained for the unit cell, which lead to increase in Purcell factor.

II. Hyperbolic Metamaterial Structure

The unit cell of the structure is shown in (Fig. 1) has an anisotropic permittivity tensor $[\varepsilon_{//}, \varepsilon_{//}, \varepsilon_{zz}]$, with the property $\mathrm{Re}(\varepsilon_{//}) \cdot \mathrm{Re}(\varepsilon_{//}) < 0$. It leads to different dispersion relations for TE ($E_z = 0$) and TM polarisations ($H_z = 0$),

$$\frac{k_x^2 + k_y^2 + k_z^2}{\varepsilon_{//}} = \frac{\omega^2}{c^2} \tag{1}$$

$$\frac{k_x^2 + k_y^2}{\varepsilon_z} - \frac{k_z^2}{|\varepsilon_{//}|} = \frac{\omega^2}{c^2} \tag{2}$$

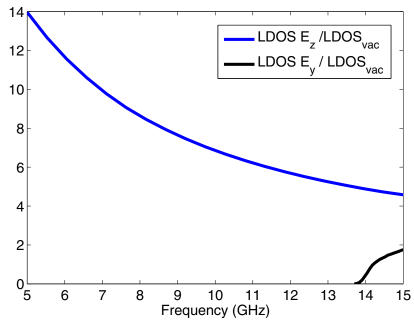

Fig. 3. Purcell factor obtained by calculation of LDOS in the periodic unit cell shown on (Fig. 1), for dipole orientations along $\hat{z}$ and $\hat{y}$.

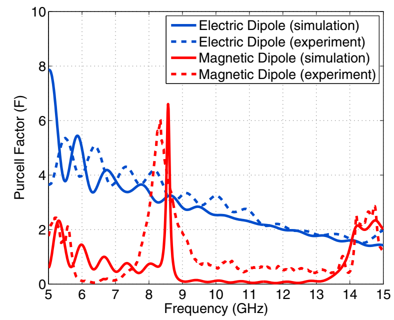

Fig. 4. Purcell factor obtained from FDTD calculations of impedance of electric and magnetic dipole antennas. The electric dipole emits in the TM polarisation and the Purcell factor is enhanced due to hyperbolic dispersion, the magnetic dipole emits in the TE polarisation and the dispersion relation does not permit propagating solutions below the plasma frequency which leads to decrease in the Purcell factor.

the hyperbolic dispersion relation in (2) leads to a large LDOS and Purcell factor (Fig. 2). The TE polarisation (1) does not have any real solutions if $\varepsilon_{//} < 0$, the Purcell factor in this polarisation is suppressed due to the unavailability of propagating modes.

III. Purcell Factor Calculation

A. Purcell factor calculation from LDOS

The Purcell factor is directly related to the change in LDOS at the location of the dipole. The LDOS at location $\mathbf{r}$ for a dipole oriented along $\hat{\mathbf{e}}_d$ can be computed from the iso-frequency surfaces of the periodic unit cell as [5]

$$N(\mathbf{r}, \omega, \hat{\mathbf{e}}_\mathbf{d})\mathrm{d}\omega = \frac{1}{(2\pi)^3}\int_{\delta\omega} \mathrm{d}k_1^{//}\mathrm{d}k_2^{//}|\hat{\mathbf{e}}_d \cdot \mathbf{E}(\mathbf{r})|^2 \frac{\mathrm{d}\omega}{|\vec{\nabla}\omega_k|} \quad (3)$$

the integration is performed over the iso-frequency surface as shown (Fig. 2). The Purcell factor is then obtained by taking the ratio of (3) to the LDOS of vacuum, shown in (Fig. 3) .

B. Purcell factor from FDTD calculations of antenna impedance

In the semi-classical approximation, the Purcell factor is same as the ratio of change in radiated power for a classical dipole antenna and is calculated from the impedance (Z). For a weakly lossy antenna the ratio of change in the real component of the antenna impedance (Z) is equal to the Purcell factor (F) [2],

$$F = \frac{\mathrm{Re}(Z)}{\mathrm{Re}(Z)_{vac}} \quad (4)$$

we calculate the Purcell factor from FDTD calculations of the impedance (Z) of electric and magnetic dipoles inside the structure. Z_{vac} is the impedance of the same antenna in vacuum. The Purcell factor is then obtained using (4), the results are presented in (Fig. 4).

IV. Conclusion

We have compared two methods for the calculation of the Purcell factor, FDTD calculations of impedance of antennas and computation of LDOS from the band diagram. We have applied the methods to analyse the Purcell factor of a hyperbolic metamaterial. We find agreement in the behaviour of the Purcell factor obtained from the two methods. From both methods we find enhancement in the Purcell factor in TM polarisation due to the hyperbolic dispersion. No propagating modes exist in the TE polarisation below the plasma frequency, which leads to a decrease in the Purcell factor when it is obtained from the impedance of the magnetic dipole.

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