Mathematical Model of Nonlinear Coupling and its Effect on Rotation Sensitivity of Semiconductor Ring Laser Gyroscope

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***Abstract*—The moving population inversion gratings formed due to spatial hole burning inside a semiconductor ring laser gyroscope causes nonlinear coupling of the counter-traveling modes. We mathematically demonstrate this phenomenon using Perturbation Theory and calculate the limit imposed by it on the sensitivity of inertial rotation.**

I. Introduction

Integrated on-chip semiconductor ring laser gyroscopes (SRLG) hold the promise of reliable, compact, low cost and low power alternative to the bulky He-Ne ring laser gyros [1]. These devices tend to reduce the overall size of the attitude and heading reference system (AHRS) of missiles, aircrafts and satellites, thus increasing their payload carrying capability and reducing the cost simultaneously [2]. Many designs of integrated SRLG have been proposed and implemented, but have failed to achieve the high performance levels required for military applications because of fairly high lock-in threshold as compared to He-Ne RLG [3]. The lock-in threshold in any gyro depends upon the strength of backscattering and coupling between the clockwise (CW) and counterclockwise (CCW) modes inside the ring laser.

In this paper, we mathematically model the nonlinear coupling between the CW and CCW waves inside a SRLG, which occurs due to backscattering from the moving population inversion gratings. Also, the effect of coupling on the rotation sensitivity of the SRLG is also analyzed. The analysis shows that, nonlinear coupling leads to frequency synchronization of the CW and CCW modes causing the suppression of beat signal and high value of lock-in threshold.

II. Mathematical Analysis

In the presence of two counter-traveling modes inside the ring laser gyro, the temporal and spatial variations of the resultant intensity causes similar variations in the carrier density leading to the formation of grating like structure as shown in Fig 1. As the gyro rotates, the gratings start moving because of the frequency difference between the CW and CCW modes. The dependence of refractive index of the semiconductor gain medium on its carrier density causes the counter-traveling modes to backscatter and they couple to each other as shown in Fig. 1 [4]. Thus, the overall electric field inside the ring cavity is given by

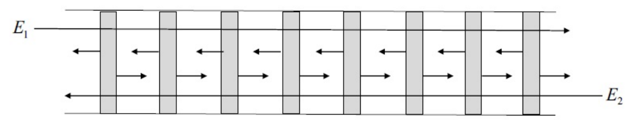

Fig. 1: Backscattering of modes from population inversion grating formed inside the semiconductor gain medium

$$\bar{E} = U(x,y)[E_f e^{j(\omega_1 t - \beta_1 z)} + E_b e^{j(\omega_2 t + \beta_2 z)}] \quad (1)$$

where $U(x,y)$ is the transverse mode distribution, E_f and E_b, ω_1 and ω_2, β_1 and β_2 are the amplitudes, frequency and wave vector of forward and backreflected wave respectively. In order to get the coupling between the forward and reflected wave, we substitute Eq. (1) into wave equation

$$\Delta^2 \bar{E} - \frac{\epsilon}{c^2}\left(\frac{\partial^2 \bar{E}}{\partial t^2}\right) = 0 \quad (2)$$

Next, we apply Perturbation Theory [5] and let the dielectric constant and optical mode to vary by small amounts by substituting $\epsilon \rightarrow \epsilon + \Delta\epsilon$ and $U \rightarrow U + \Delta U$ in Eq. (2). While solving for the time varying coupling effects, we neglect the terms that lead to unperturbed wave equation and consider only the first order perturbation terms, dropping the higher order perturbations. Also, to focus on the time varying effects of rotation of a gyro on electric field, we consider only the temporal derivatives. The coupled equations are then given as

$$\frac{dE_f}{dt} = \frac{-j\beta_2^2 c^2}{2\omega_1} E_b e^{j(\omega_2-\omega_1)t} e^{-j(2\pi/T)t} \frac{\int \delta\epsilon(x,y)|U|^2 dA}{\int(\epsilon+\delta\epsilon)|U|^2 dA} \quad (3)$$

$$\frac{dE_b}{dt} = \frac{-j\beta_1^2 c^2}{2\omega_2} E_f e^{j(\omega_1-\omega_2)t} e^{-j(2\pi/T)t} \frac{\int \delta\epsilon(x,y)|U|^2 dA}{\int(\epsilon+\delta\epsilon)|U|^2 dA} \quad (4)$$

where the variations in dielectric constant are assumed to be sinusoidal with a time period T i.e.

$$\Delta\epsilon = \frac{\delta\epsilon(x,y)}{2} e^{j(2\pi t/T)} + \frac{\delta\epsilon(x,y)}{2} e^{-j(2\pi t/T)} \quad (5)$$

Now, assuming perfectly normalized transverse modes and expressing the dielectric variations in terms of group refractive

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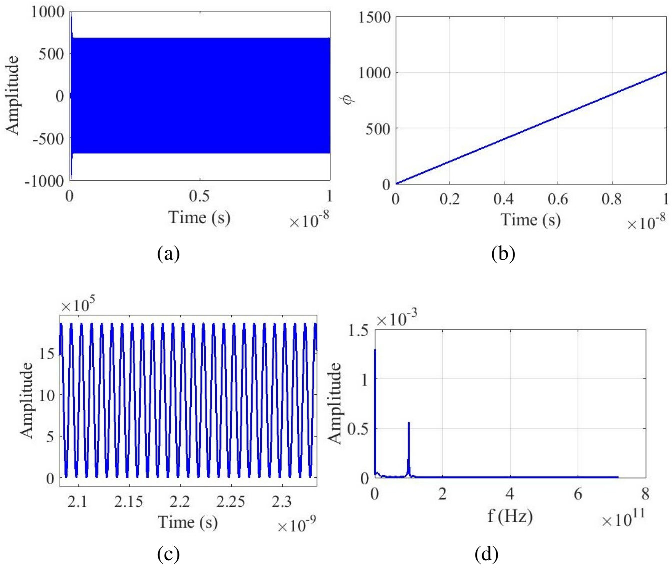

Fig. 2: (a) Electric field distribution, (b) instantaneous phase, (c) time domain beat signal, (d) frequency domain beat signal for no lock-in case i.e. $\Omega_r >> \Omega_L$

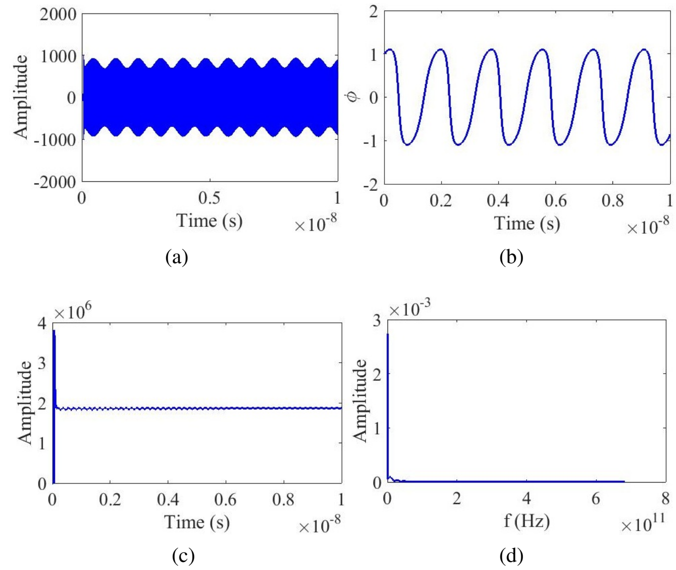

Fig. 3: (a) Electric field distribution, (b) instantaneous phase, (c) time domain beat signal, (d) frequency domain beat signal for lock-in case i.e. $\Omega_r < \Omega_L$

index variations (δn_g) as $\delta\epsilon(x, y) = 2n_g\delta n_g(x, y)$, we get the magnitude of generalized coupling coefficient (κ) from Eq. (3) and (4) as

$$\kappa = \frac{\beta^2 c^2}{2\omega}\left(\frac{n_g\delta n_g(x, y)}{\epsilon + n_g\delta n_g(x, y)}\right) \tag{6}$$

The rate equations of the CW and CCW electric fields inside an integrated SRLG considering the nonlinear coupling can be written as [6]

$$\frac{dE_{1,2}}{dt} = v_g(g_c - \alpha_i)E_{1,2} - j(\omega_{1,2} - \Omega) - j\kappa E_{2,1} \tag{7}$$

where v_g is group velocity, g_c is gain coefficient, α_i is average internal loss and Ω is cold cavity resonance frequency of the ring laser. Substituting $E_1 = Ae^{-j\phi_1}$, $E_2 = Ae^{-j\phi_2}$ in Eq. (7) and separating into real and imaginary parts, instantaneous frequency difference is given as

$$\frac{d\phi}{dt} = (\omega_2 - \omega_1) - j\kappa sin(\phi) \tag{8}$$

The above equation shows the instantaneous beat frequency of the rotating SRLG in the presence of nonlinear coupling of CW and CCW modes. When expressed in terms of gyro scale factor (S) and external rotation rate (Ω_r), Eq. (8) becomes

$$\frac{d\phi}{dt} = 2\pi S\Omega_r - j\kappa sin(\phi) \tag{9}$$

Lock-in threshold of SRLG, which is the value of rotation rate below which the instantaneous frequency difference becomes zero, is given from Eq. (9) as

$$\Omega_L = \frac{\kappa}{2\pi S} \tag{10}$$

III. Simulation Results and Conclusions

Fig. (2) and (3) show the comparison of the electric field distribution ($\bar{E}$), instantaneous phase (ϕ) and output of SRLG in time and frequency domain when the gyro is operating in lock-free regime ($\Omega_r >> \Omega_L$) and locked regime ($\Omega_r < \Omega_L$). As seen from Fig. (3c) and (3d), the beat signal is completely lost in the locked zone of the gyro.

In conclusion, nonlinear backscattering from the population inversion grating and the resultant coupling of counter-traveling waves imposes a severe limitation on the operation of integrated SRLG. For a typical on-chip SRLG, the lock-in threshold as calculated from Eq. (10) can be as high as $1\times10^{8}\ ^{0}/h$, thus making it unsuitable for use in military applications. In order to achieve high performance from SRLG, the nonlinear coupling needs to be eliminated by use of methods such as orthogonal polarization of counter-traveling waves and external biasing using a phase modulator.

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