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Electronic structure of QD arrays: Application to intermediate-band solar cells

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**Design of the solar cell with
POWER conversion efficiency >50%
with cost of <\$100 per 1m² of panel !!!**

**This will make PV solar cell economically
competitive**

**Today: \$0.25-\$0.65/kWh from PV vs.
\$0.04/kWh from coal**



Performance of photovoltaic and photochemical solar cells

Type of cell	Efficiency (%)*		Research and technology needs
	Cell	Module	
Crystalline silicon	24	10-15	Higher production yields, lowering of cost and energy content
Multicrystalline silicon	18	9-12	Lower manufacturing cost and complexity
Amorphous silicon	13	7	Lower production costs, increase production volume and stability
CuInSe ₂	19	12	Replace indium (too expensive and limited supply), replace CdS window layer, scale up production
Dye-sensitized nanostructure materials	10-11	7	Improve efficiency and high-temperature stability, scale up production
Bipolar AlGaAs/Si photochemical cells	19-20	-	Reduce material cost, scale up
Organic solar cells	2-3	-	Improve stability and efficiency

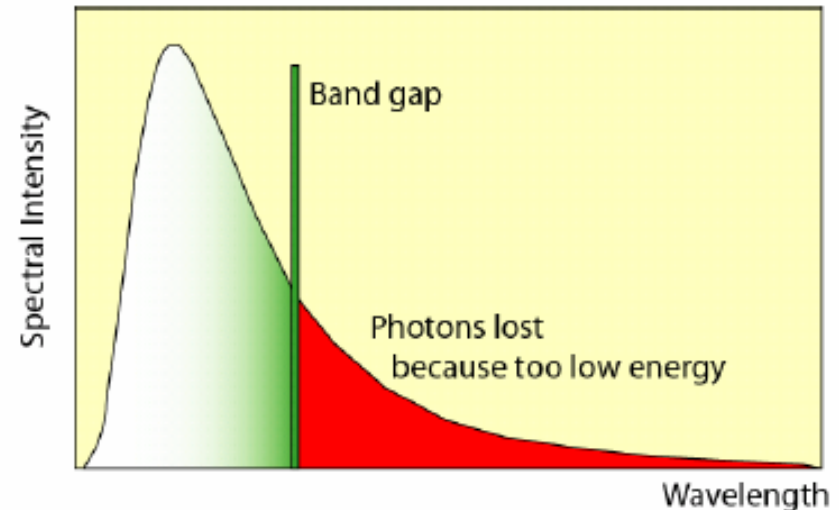
M. Grätzel, Nature 415, 338 (2001)



- Thermodynamic limit of Carnot engine: $\eta = 1 - T_0/T_s \sim 95\%$ (100% absorption)
- Thermodynamic limit of solar heat engine: $\eta = (1 - T_a^4/T_s^4)(1 - T_0/T_s) \sim 85\%$
- Shockley-Queisser efficiency limit for single band semiconductor based on detail balance eq.:
~31% (1 sun: Planck low) and ~41 (max conc.)

Origin of the solar cell losses:

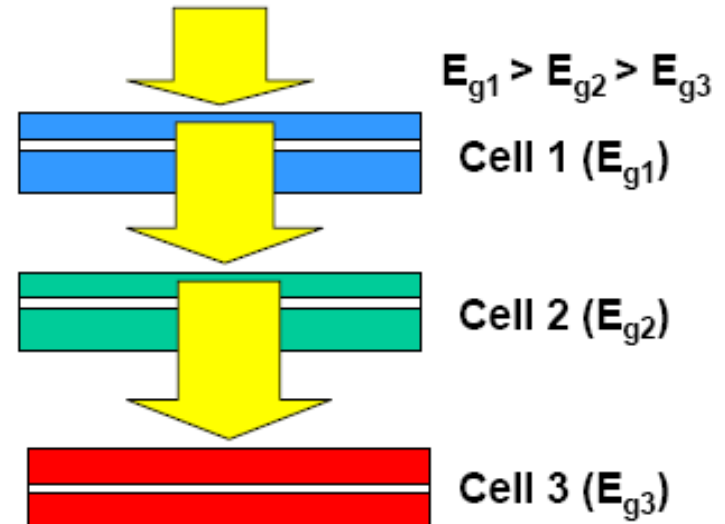
- a) Light with energy below E_g will not be absorbed
- b) The photons with excess energy above E_g is lost in the form of heat
- c) Single crystal GaAs solar cell $\sim 25\%$ (AM1.5)



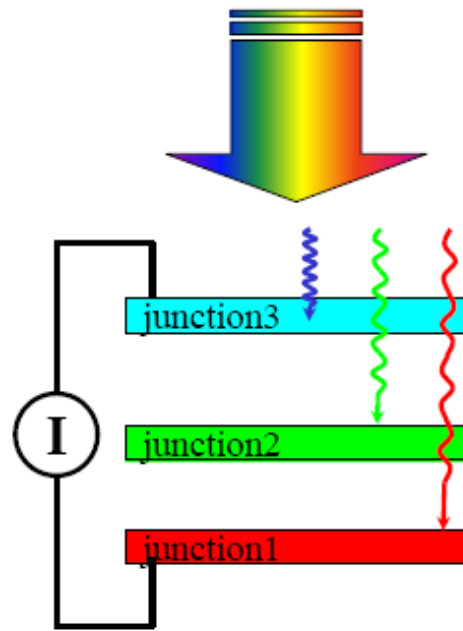


Multijunction or tandem cells:

- First approach to exceed single junction efficiency
- To achieve >50% efficiency need 3 or more tandems with different E_g 's
- Significant technological problem to relax strain
- 75% efficiency achieved with 36 tandems

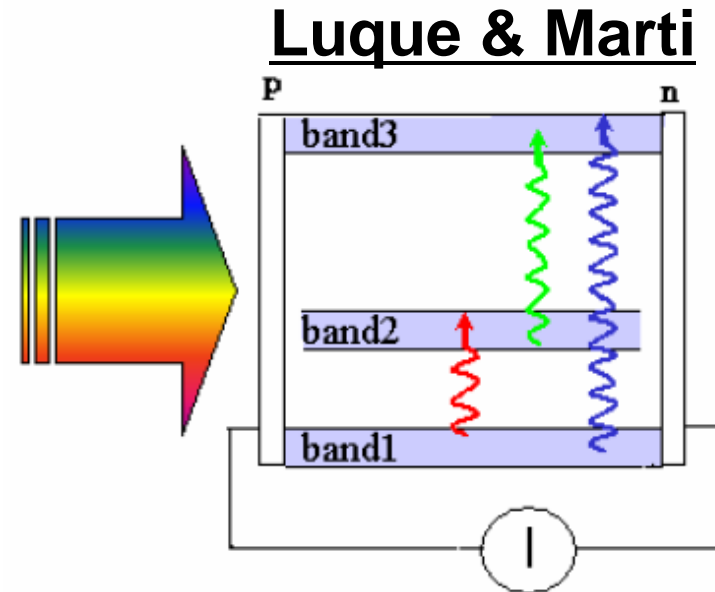


No of junctions	1 sun	Max conc.
1	30.8%	40.8%
2	42.9%	55.7%
3	49.3%	63.8%
∞	68.2%	86.8%



Multi-junction solar cell

- Each junction \Rightarrow single gap
- N- junctions \Rightarrow N- absorptions

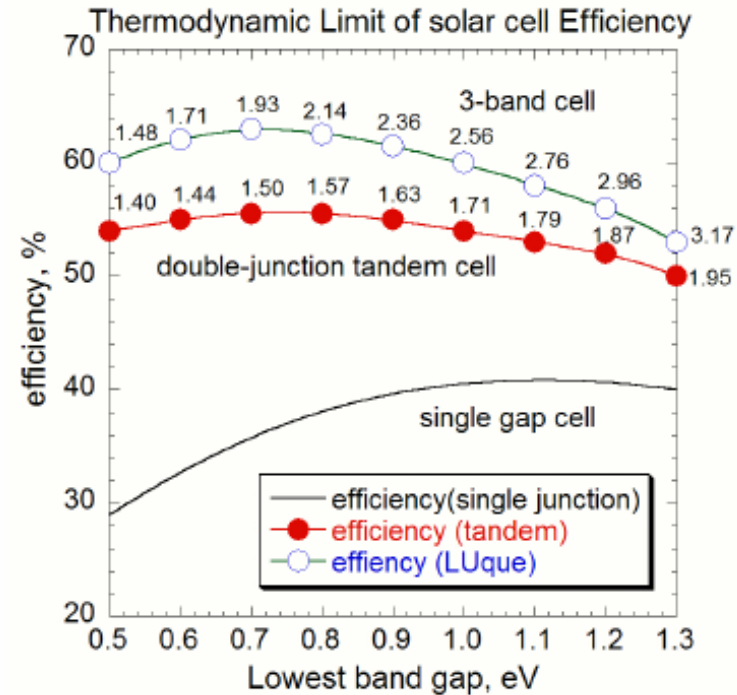
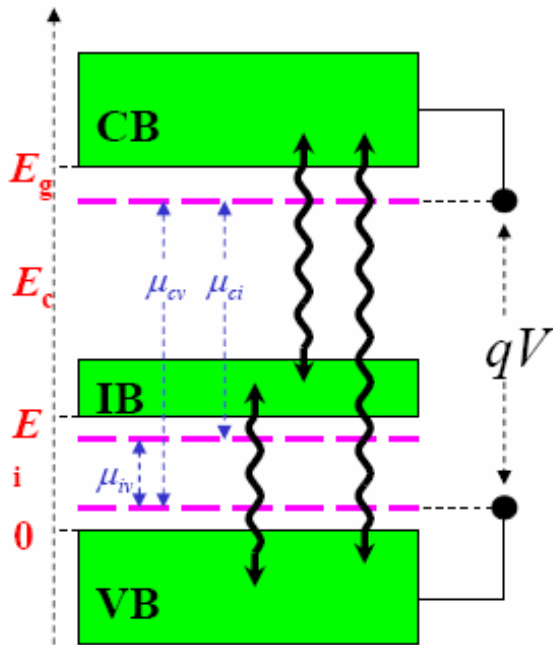


Multi-band solar cell

- Single junction (no lattice mismatch)
- N- bands \Rightarrow $N(N-1)/2$ (gaps) absorptions
- Add 1 band \Rightarrow Add N- absorptions



Intermediate band solar cells



Intermediate band vs multi-junction solar cell

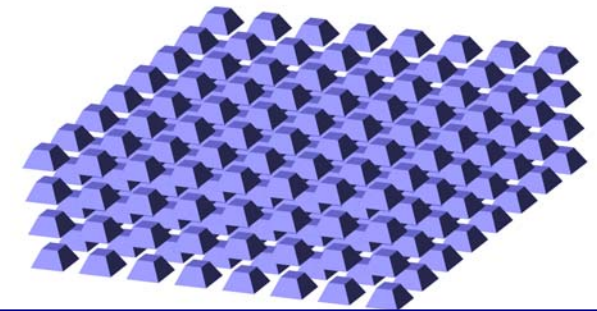
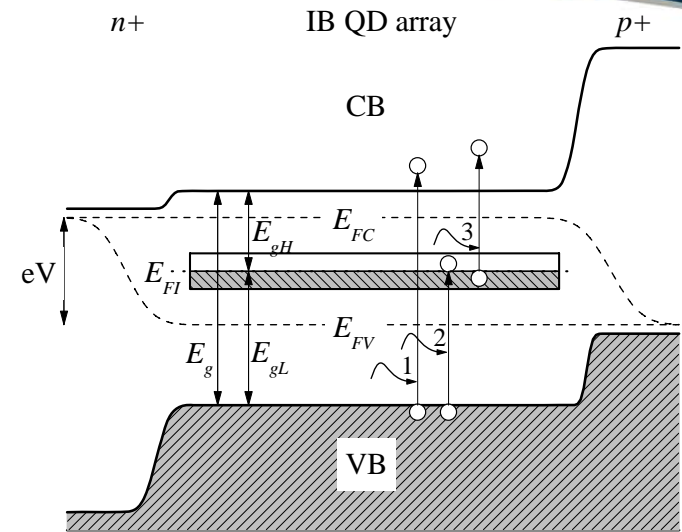
- Max. efficiency for 3 band cell ~66% (vs 55%)
- Max. efficiency for 4 band cell ~72% (vs 60%)
- Better performance than any other structure of similar complexity



There are two **contradictory requirements** for IB:

(a) the IB should exhibit finite energy width so that it can be partially occupied and facilitate simultaneous excitation from IB to CB and VB to IB

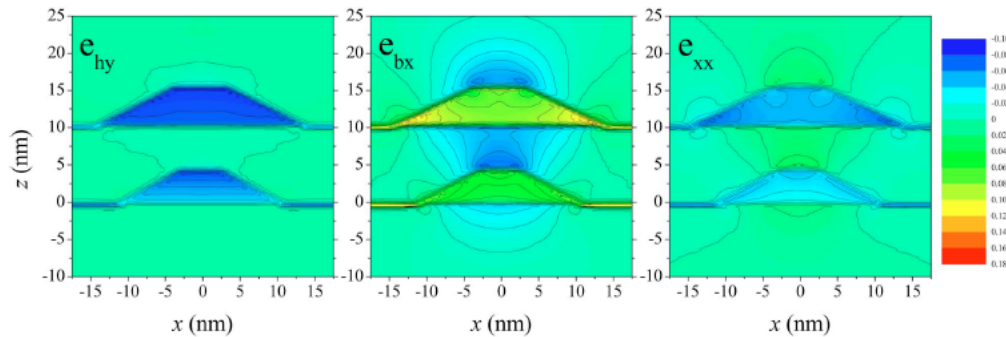
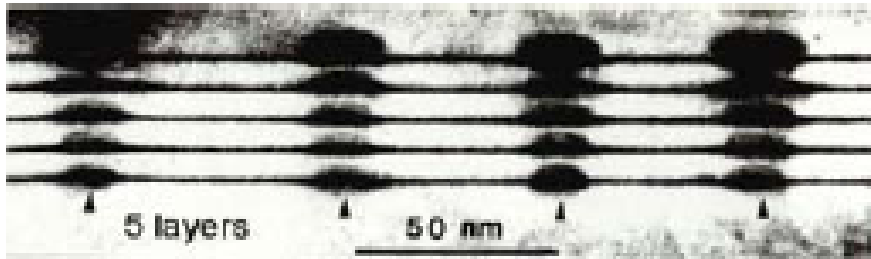
(b) the IB should be as narrow as possible to reduce carrier transport through the mini-band.



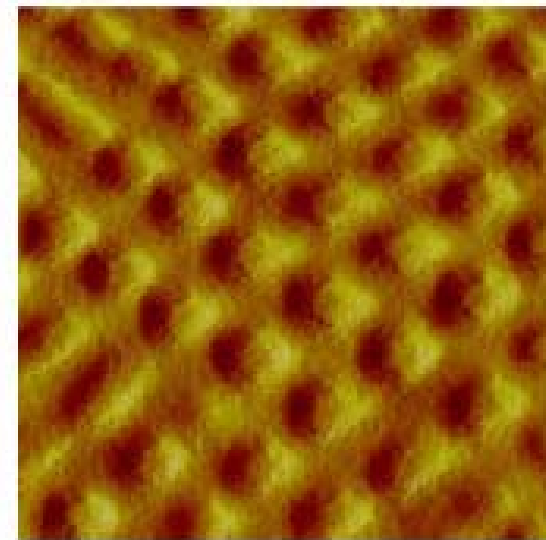
Arrays of QD ideal candidate for realisation of IB because of zero density of states between VB & IB & CB that increase radiative lifetime relative to relaxation time within bands



InAs QDs on GaAs(100)



InAs quantum dots



(500nm x 500nm)

- Vertical ordering is provided by strain driven alignment
- Horizontal regularity of QD's is observed on high Miller indices growth (311)

Q. Xie, et al., Phys. Rev. Lett. **75**, 2542 (1995)

S. Tomic et al., J. Appl. Phys. **99**, 093522 (2006)

Y. Okada, private communication



Fourier transform of the strain tensor in analytical form for the cubic crystal symmetry and actual truncated-pyramid QD shape

Analytical expressions for the Fourier transform of the piezoelectric field

Plane Wave expansion (Fourier) method for the QD carrier spectra and wave functions from 8 band kp theory

Use of Hamiltonian C2 symmetry property: $\sim 2 \times \text{rank}(4)$ diagonalizations instead $\sim \text{rank}(8)$ diagonalization (8 times sped up)

$H = H_k + H_e + V_{pz}$ is dense matrix \Rightarrow **NOT sparse**

Direct diagonalization by ScaLAPACK and MPI packages on HPCx

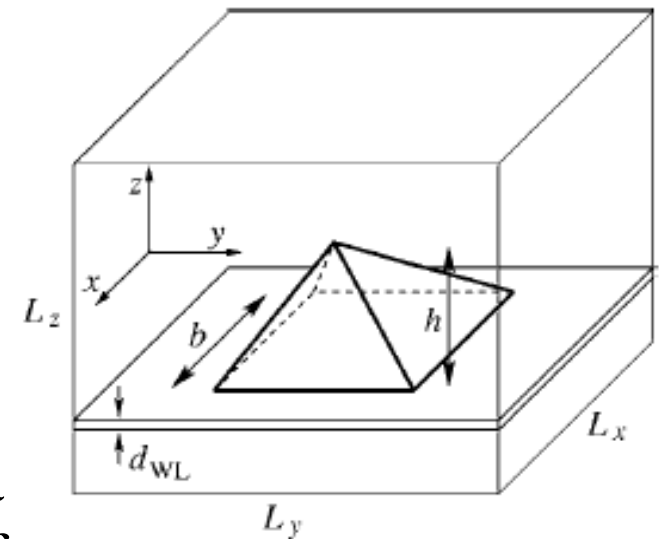


To calculate the electronic structure of an QD array the only modification to the basis set is

$$\mathbf{k}_v \rightarrow \mathbf{k}_v + \mathbf{K}_v^{SL} \quad v \in (x, y, z)$$

$$\mathbf{K}_v^{SL} = a(\pi/L_v^{SL}) \quad a \in [0, 1]$$

This allows the sampling along the K points of a QD-SL to be done at several points at the cost of the single QD calculation at each K point.





Scalability

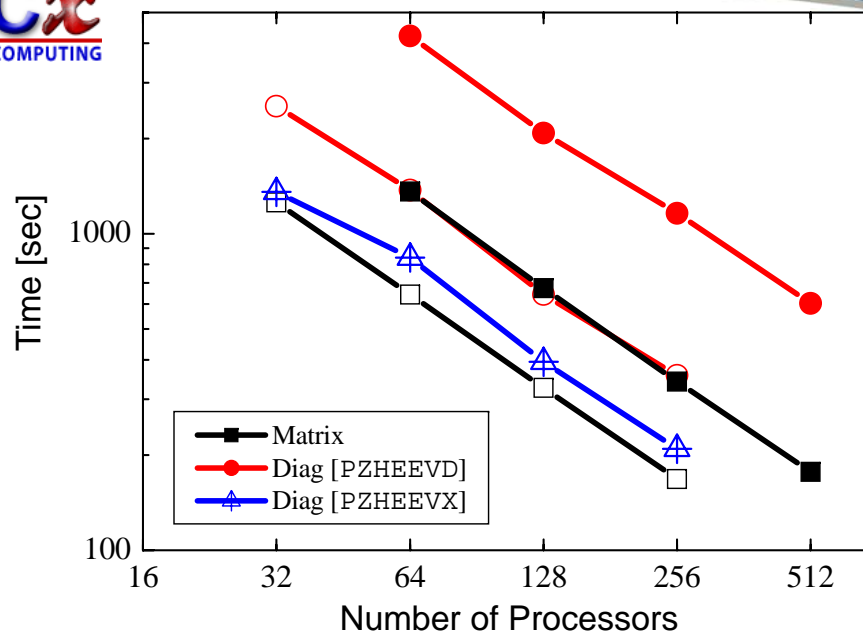


www.top500.org

Rank 65

- 160 IBM Power5 p5-575SMP nodes
- 2560 processors
- 15.3 TeraFlop/s peak
- 12.9 TeraFlops/s sustained
- 5.12 TByte of memory

Located at STFC Daresbury Laboratory



$N = (6,6,9) = 25688$ (from 32 to 256)

Scalability = 1.93^3 (ideal 2^3)

$N = (7,7,10) = 37800$ (from 64 to 512)

Scalability = 1.93^3 (ideal 2^3)



Truncated pyramid QD shape

$$b = 120 \text{ \AA}$$

$$h = 60 \text{ \AA}$$

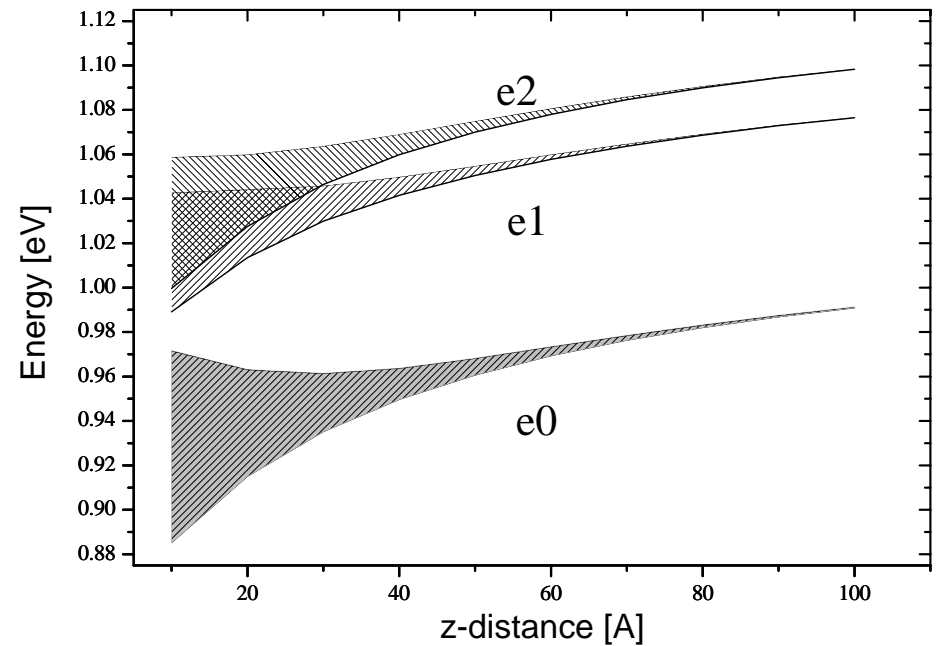
$$t = 0.5$$

$$L_x = L_y = 200 \text{ \AA}$$

vertical distance:

$$d_z = L_z - h \in [10, 100] \text{ \AA}$$

material: InAs/GaAs



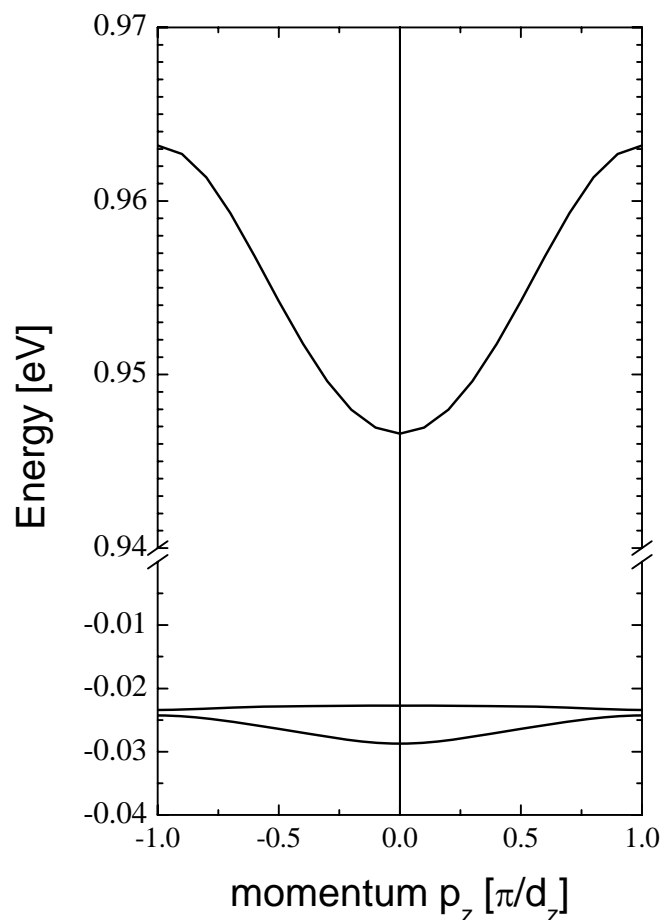
$$\text{band width} = E(K_z = \pi/L_z) - E(K_z = 0)$$

$$\text{e0 width: } 86 \text{ meV @ } d_z = 10 \text{ \AA}$$

$$< 1 \text{ meV @ } d_z = 100 \text{ \AA}$$

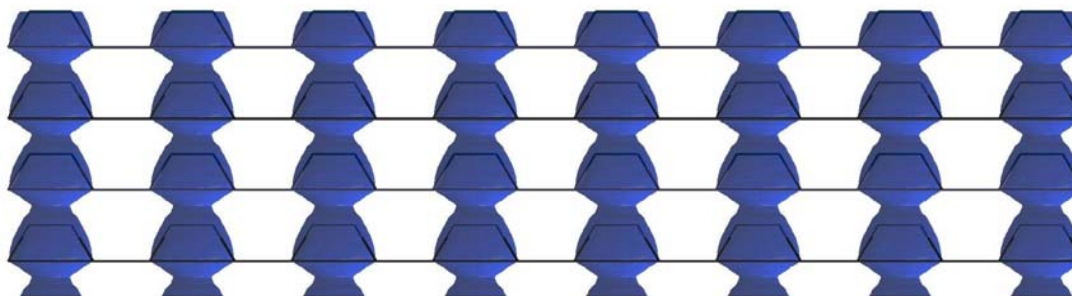


Electronic structure of IB



Electronic structure of 40 Å vertically spaced QD array:

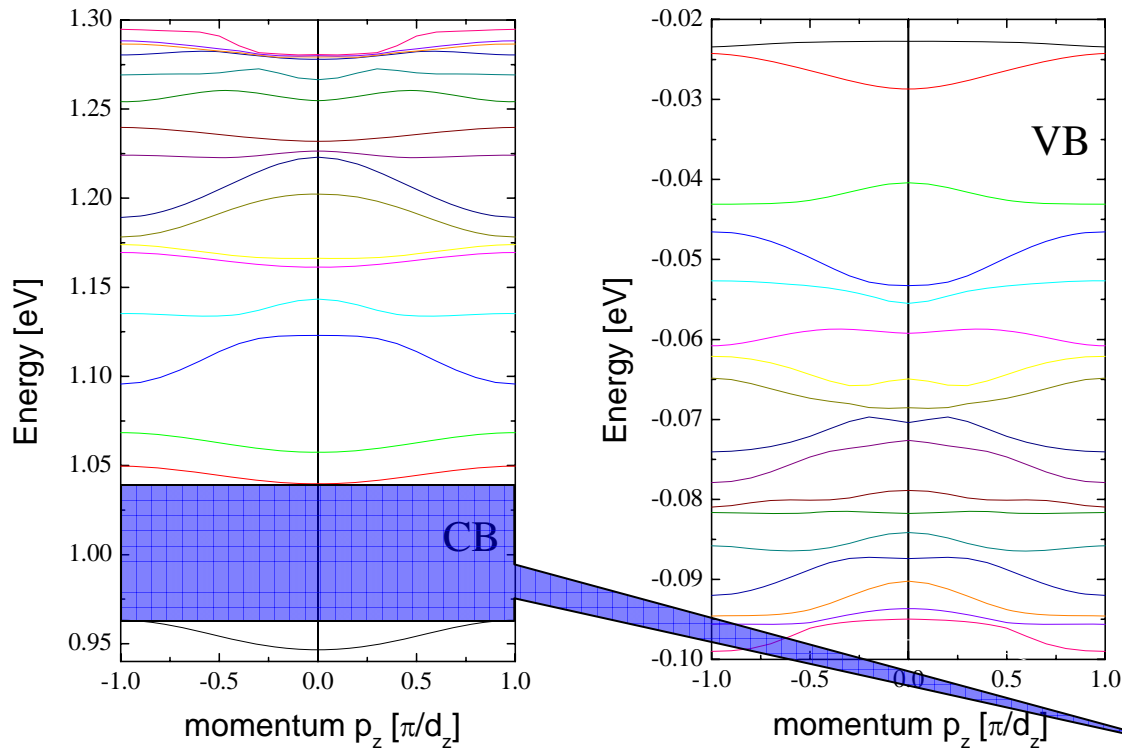
- e_0 width 14 meV @ $d_z = 40$ Å
- h_0 & h_1 almost dispersionless





Electronic structure of IB

$$d_z = 40 \text{ \AA}$$



In VB probably all minibands will merge because of small energy separation $< \sim kT$ (25 meV @ 300K)

In CB probably just one miniband : e_0

In CB probably just 1 gap

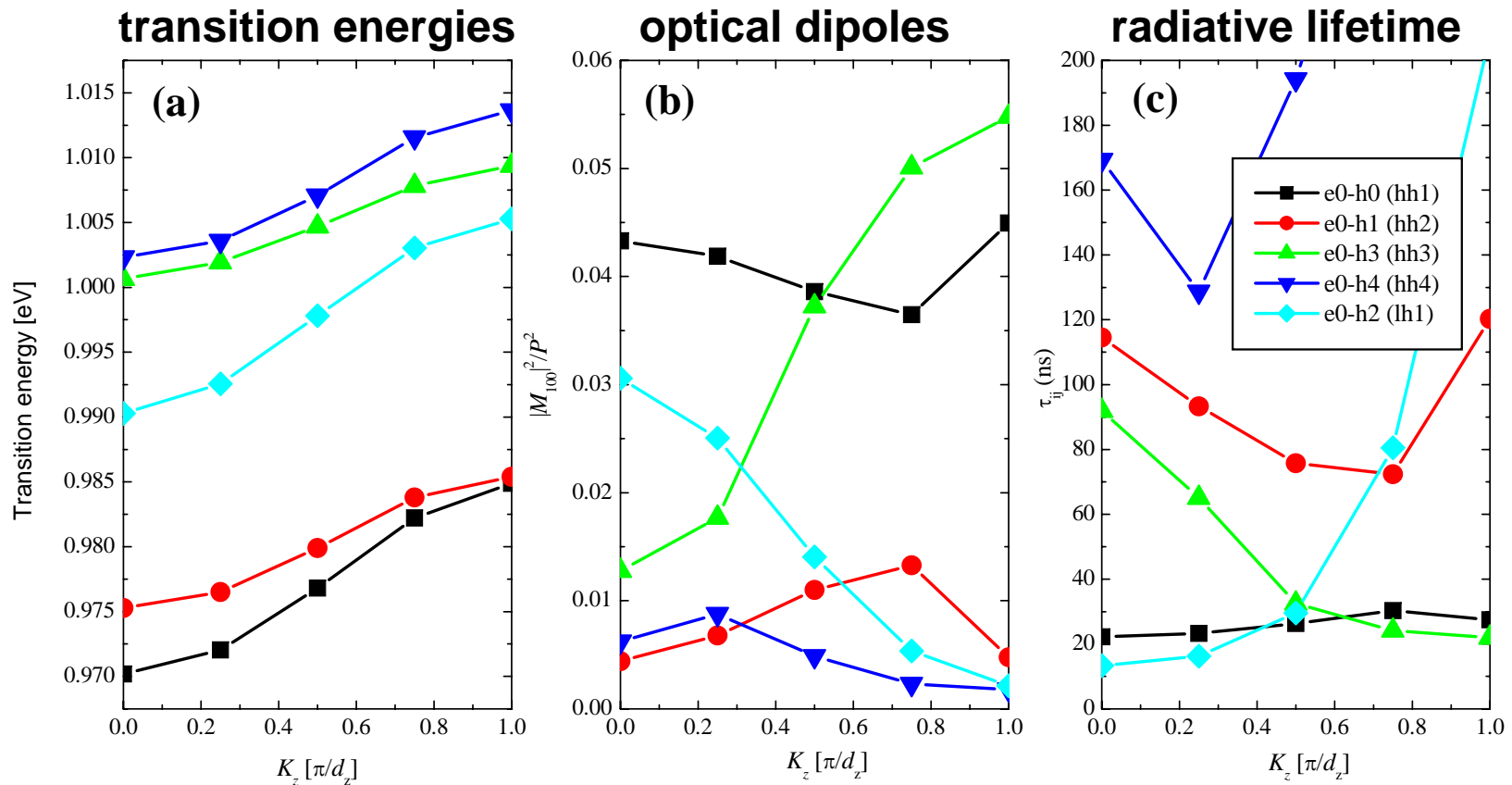
First Brillouin zone



Fermi Golden rule:

$$\frac{1}{\tau_{if}^{\text{rad}}} = \frac{(E_i - E_f) \bar{n}}{3\pi\epsilon_0 \hbar^2 c^3} (M_x^2 + M_y^2 + M_z^2)$$

~25 ns in QD-SL vs. ~2 ns in QD





- ❑ **8 band kp theory can successfully predict electronic and optical structure of QD arrays**
- ❑ **PW methodology with periodic boundary condition is particularly suited for QD array analysis**
- ❑ **Results on radiative lifetime in QD arrays show significant increase due to existence of the IB**
- ❑ **Model presented provides much realistic parameters for drift-diffusion equations and efficiency prediction**

Future work should:

- 1) Identify relative role of the intermediate band: width, energy position, doping, etc.**
- 2) Identify best material combination**
- 3) Identify relative role of the structure/size imperfection**