On The Fractional Order Model of EDFAs With ASE

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Outline

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  - EDFAs In Fiber-optic Telecommunications
- Light Amplification in EDFAs
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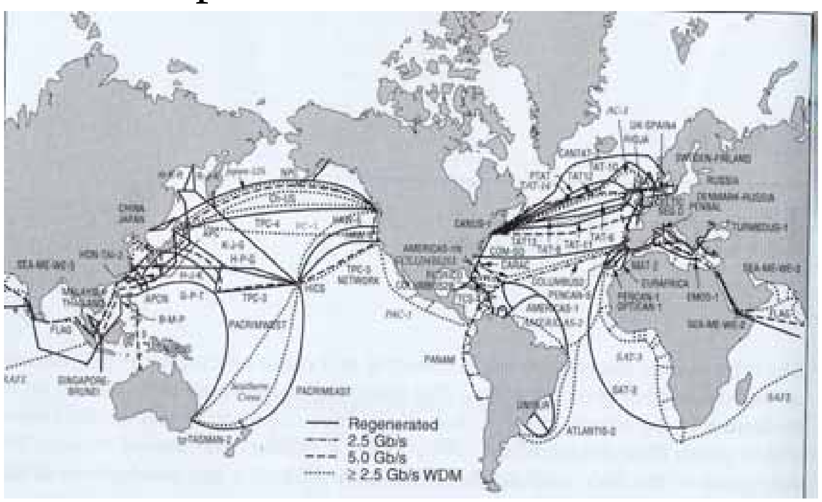
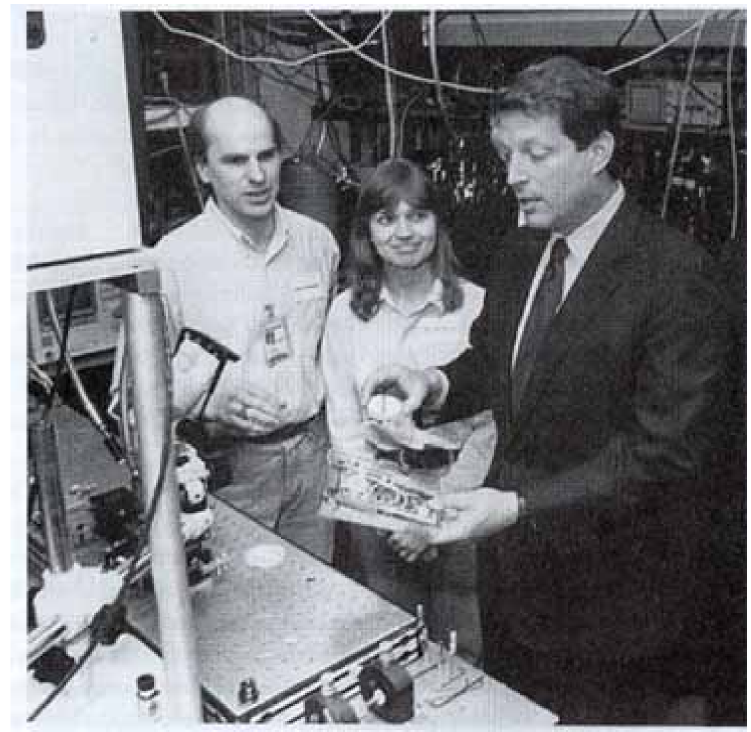
  - Experimental vs. Simulation Results
- ASE And Its Influence on EDFA's Gain
- Modeling Results Using Fractional Derivatives
- Conclusions

EDFAs in Fiber-Optic Telecommunications

EDFA is one of the most commonly used type of fiber amplifiers in both long-haul and metro optical networks.

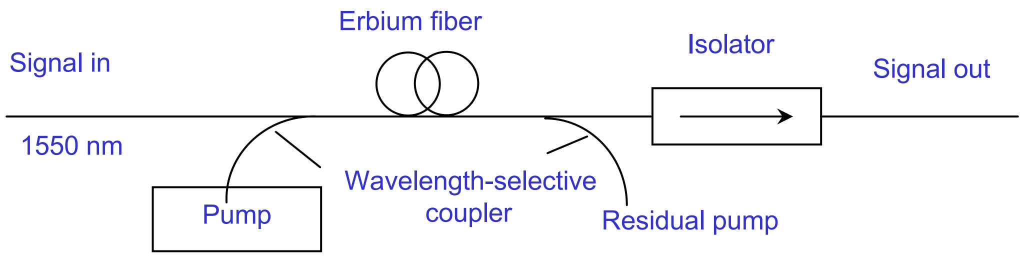




* P.C. Becker et. al. *Erbium-Doped Fiber Amplifiers: Fundamentals and Technology*, Academic Press, 1999.

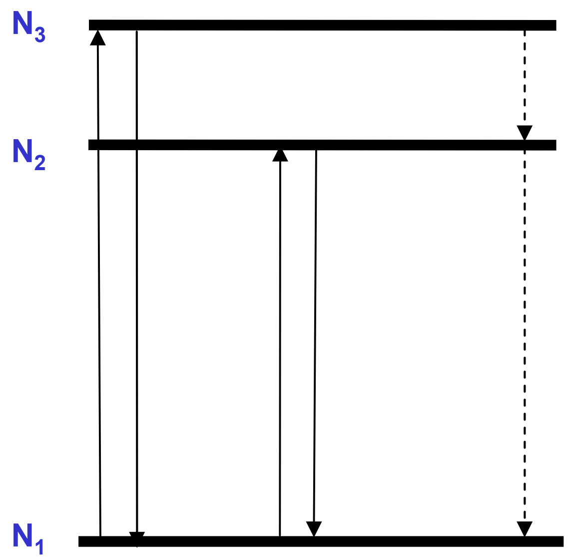
** AT&T Photo Archives

EDFA – Physical Model



Excitation from level 1 to 3 is proportional to ion populations (N_1 and N_3, respectively) in these levels

Excitation from level 1 to 2 is proportional to ion populations (N_1 and N_2, respectively) in these levels



Excitation Levels Of Erbium Ions

Spontaneous transitions from the excited levels to the ground level are proportional to the inherent parameters of EDFA called *transition probabilities* (*$Gamma_{32}$* and *$Gamma_{21}$* for spontaneous transitions from the level 3 to level 2 and from level 2 to level 1, respectively)

Mathematical Model of Light Amplification in EDFAs

Model with three temporal and two spatial differential equations (Becker et. al. 1999)

$$\frac{dN_1}{dt} = \frac{N_2}{T_{21}} - (N_1 - N_3) \cdot P_p \cdot A_p + (N_2 - N_1) \cdot P_s \cdot A_s$$

$$\frac{dN_2}{dt} = -\frac{N_2}{T_{21}} + \frac{N_3}{T_{32}} - (N_2 - N_1) \cdot P_s \cdot A_s$$

$$\frac{dN_3}{dt} = -\frac{N_3}{T_{32}} + (N_1 - N_3) \cdot P_p \cdot A_p$$

$$\frac{dP_s}{dz} = (N_2 - N_1) \cdot P_s \cdot K$$

$$\frac{dP_p}{dz} = (N_3 - N_1) \cdot P_p \cdot K$$

Physical Constraints

- Number of ions on level E_2 must exceed the number of ions on level E_1
- Energy conservation is preserved (the sum of all derivatives is zero)

Mathematical Model of Light Amplification in EDFAs

$$T_{21} = 10^3 T_{32}$$ → *two-level model approximation*

Reservoir of ions r(t) is defined as the total number of excited ions in the amplifier

$$r(t) = \rho A \int_0^L N_2(z,t)\,dz$$

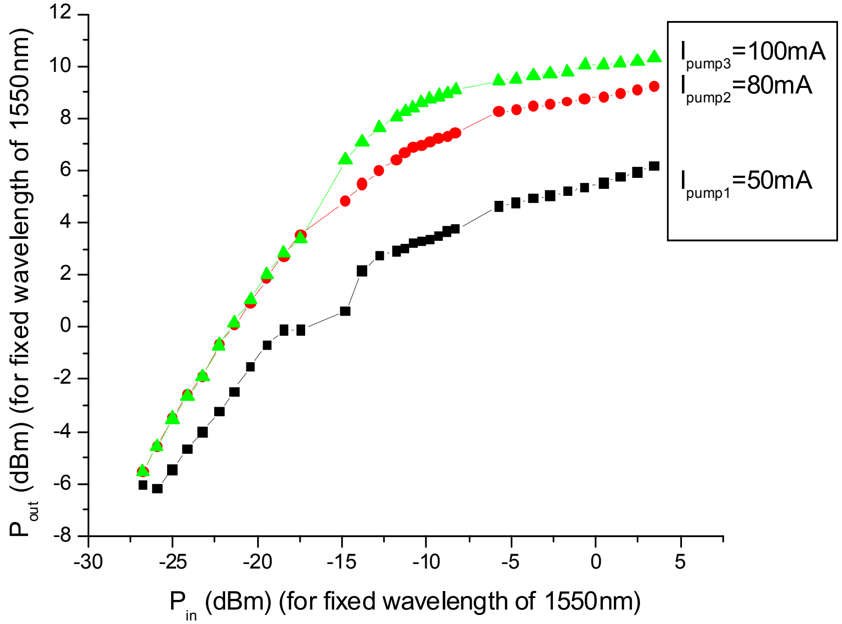
Integration of pump and source powers over length of fiber yields an ODE over N_2. Using the definition of photon fluxes the following absorption/emission model of ions is obtained*

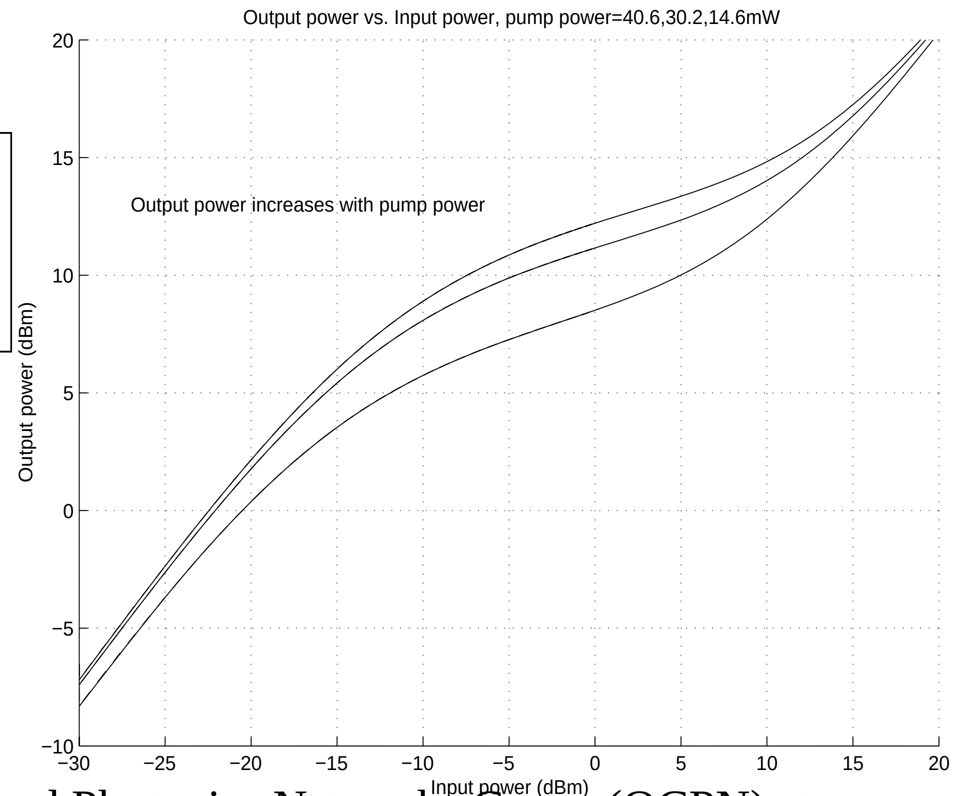
$$\dot{r}(t) = -\frac{r(t)}{\tau} + \sum_{j=0}^{N} Q_j^{in}(t)\left[1 - e^{B_j r(t) - A_j}\right]$$

***Bononi, Alberto; Rusch, Leslie A: Doped-Fiber Amplifier Dynamics: A System Perspective, *Journal of Lightwave Technology, vol. 16, no. 5,* IEEE, 1998.**

Experimental Results vs. Simulation

- Experimental efforts* suggested *saturation* phenomenon occuring in the amplifier; proven to be incorrect by simulation results

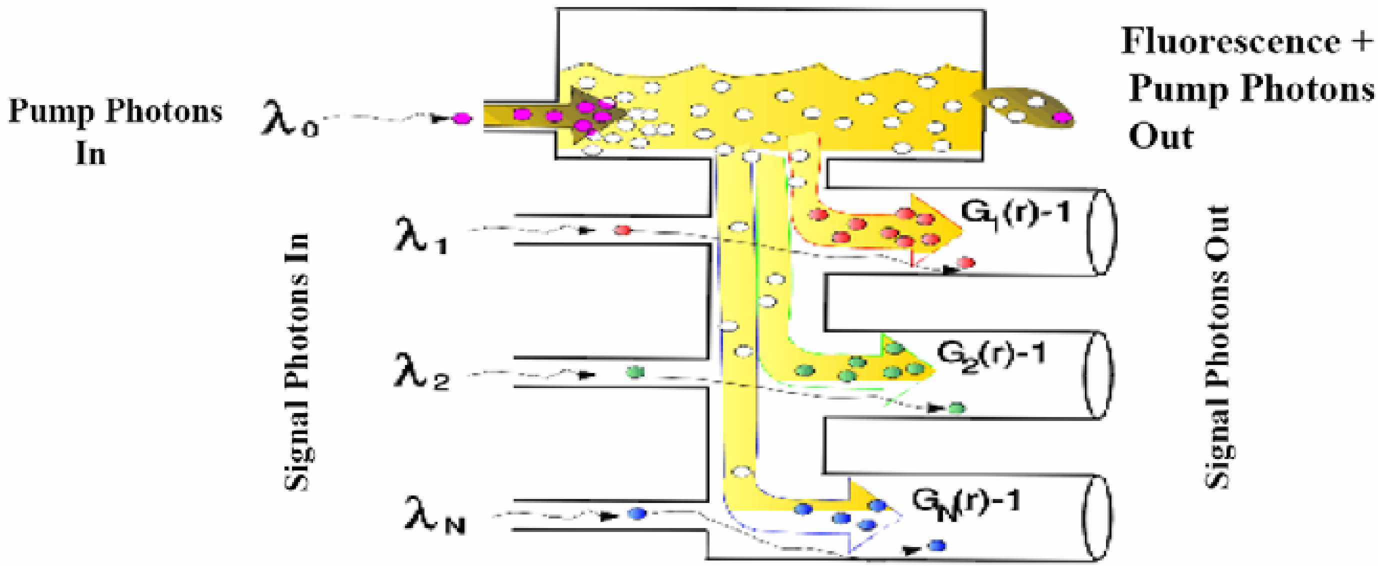




* Performed at Optical Communications and Photonics Networks Group (OCPN) at UCSB

Amplified Spontaneous Emission In EDFAs

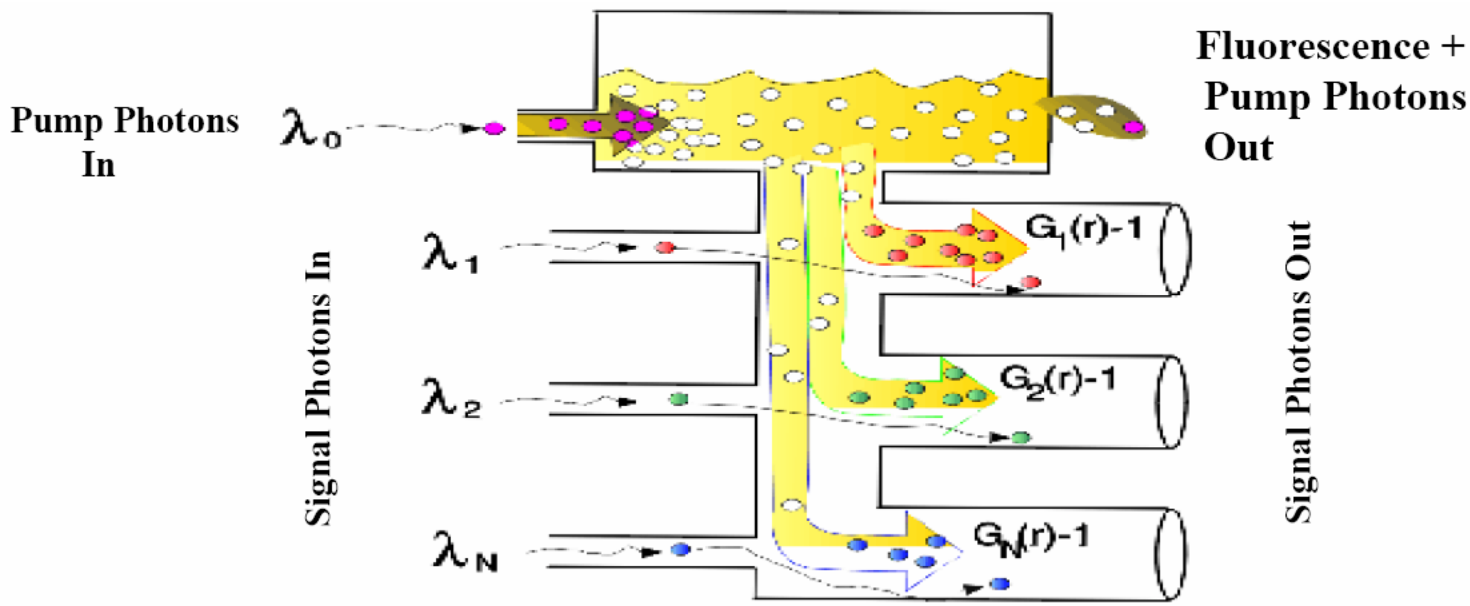
Reservoir r, Total number of excited ions



- EDFA can be observed as a "hydraulic system", composed of a reservoir of charges (excited Erbium ions), which are ready to be converted into output photons by stimulated emission; reservoir is limited to the maximum number of excited ions in fiber
- All signals draw from the same reservoir of charges, number of ions drawn is G_i-1
- This phenomenon is called homogeneous broadening

Amplified Spontaneous Emission In EDFAs

Reservoir r, Total number of excited ions



- Oscillations in reservoir occur since the average number of input photons per second (signal photon fluxes) abruptly vary in time due to sudden addition of new or removal of existing channels (analogy: oscillations of water basin)
- Lower reservoir level causes variations of the gain and lower output photon fluxes, which is known as cross-gain modulation
- Also, as do most reservoirs, this one has leakage that is formed mostly due to process of fluorescence

EDFA Model With ASE

- Amplified spontaneous emission is another effect that occurs in fiber amplifiers that was not explicitly described in the above model
- *Stimulated* emission, which exploits the avalanche effect along the fiber core, provides the gain of the amplifier
- *However*, this gain needs to be lowered due to *fluorescence* (natural relaxation of ions); these ions still produce photons, creating EDFA's optical noise (ASE) (leakage of the reservoir)

Mathematical model of absorption/emission of ions in EDFA with ASE*

$$\dot{r}(t) = -\frac{r(t)}{\tau} + \sum_{i \in \{S,A\}} Q_i^{in}[1 - G_i[r(t)]] - \sum_{i \in A} 4n_i^{sp}[r(t)][G_i[r(t)] - 1]\Delta\nu_i$$

* **Karásek, Miroslav; Bononi, Alberto; Rusch, Leslie A; Menif, Mourad: Gain Stabilization in Gain Clamped EDFA Cascades Fed by WDM Burst-Mode Packet Traffic, *Journal of Lightwave Technology, vol. 18, no. 3,* IEEE, 2000.**

EDFA Model With ASE
Control Systems Perspective

$$\dot{r}(t) = -\frac{r(t)}{\tau} + \sum_{i\in\{S,A\}} Q_i^{in}[1 - G_i[r(t)]] - \sum_{i\in A} 4n_i^{sp}[r(t)][G_i[r(t)] - 1]\Delta\nu_i$$

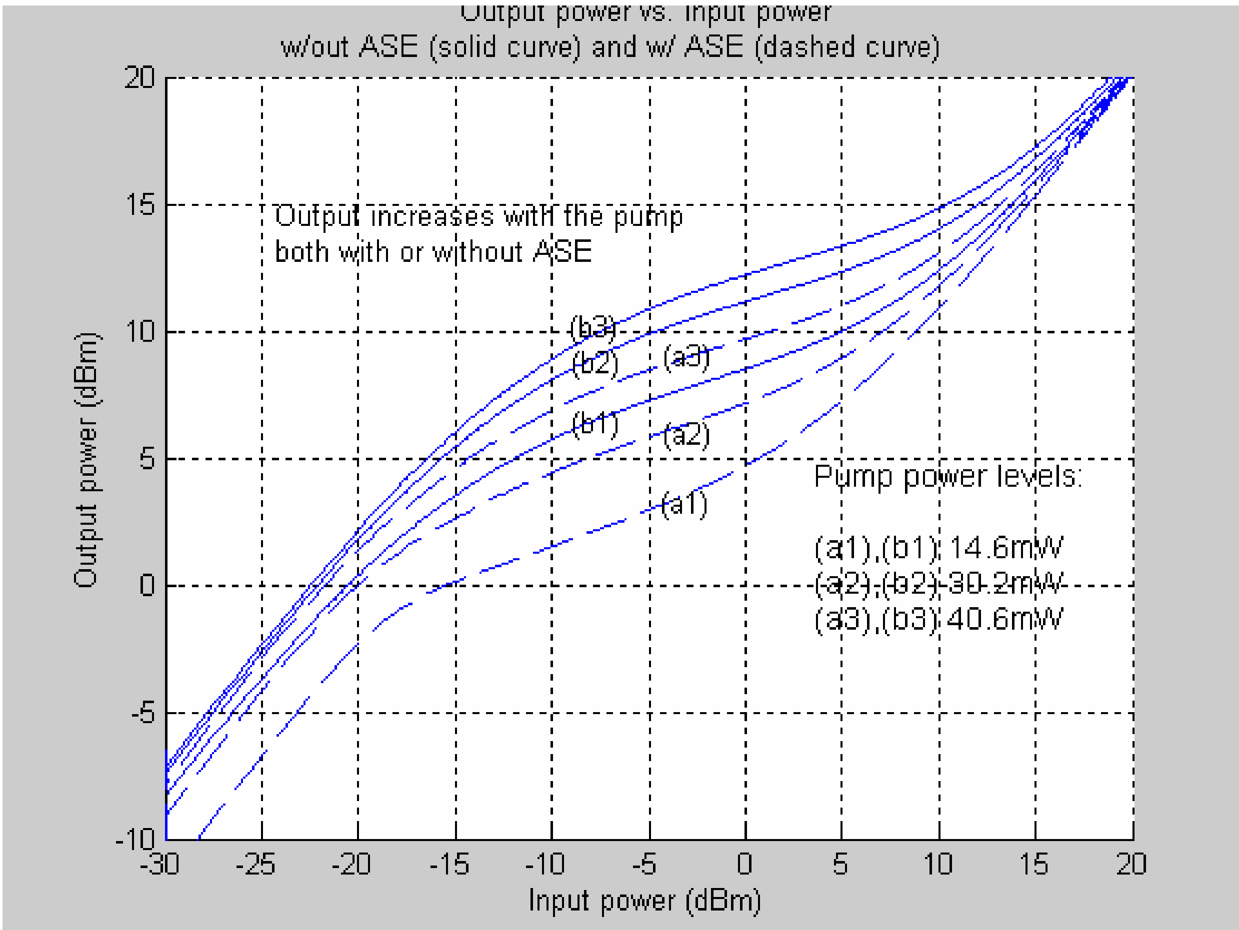
- Nonlinear equation, due to a nonlinear gain of EDFAs: $G_i[r(t)] = e^{B_i x - A_i}$
- Control System's perspective: Nonlinear system with at least two physical inputs (a pump and an input signal), with pump power Q_p that can be manipulated up to an extent
- Use input pump flux as a control input u
- System is intrinsically stable
- Simplified system without ASE looks as follows

$$\dot{x}_1 = -\frac{1}{\tau}x_1 + x_2(1 - e^{B_p x_1 - A_p}) + 20d(1 - e^{B_s x_1 - A_s})$$

$$\dot{x}_2 = u$$

ASE Influence On Gain of EDFA

Influence of ASE on the overall gain of EDFA is shown in the following graph



Gain is decreased when ASE is included!

Fractional Derivatives In Modeling Of EDFAs

Idea: To model memory effect of ASE with fractional derivative i.e.

$$x^{(\alpha)} = f(t,x) \quad 0<\alpha<1$$ **Bonnoni equation**

$$x^{(1)} = f(t,x) + g_{ASE}(t,x)$$

Nonlinearity due to ASE

Motivation comes from the classical theory of fractional derivatives, where it is applied on the elements with memory (avalanche) effects

Most common definition of fractional derivatives

Riemann-Liouville $0<\alpha<1$

$$\left({}_0D_t^{\alpha}x\right)(t) = \frac{d}{dt}\frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{x(\tau)}{(t-\tau)^{\alpha}}d\tau \qquad 0<\alpha<1$$

Note: $x^{(\alpha)} \to \infty$ when $t \to 0$ if $x(0) \neq 0$

So, this definition used only with non-zero initial conditions

Fractional Derivatives In Modeling Of EDFAs

Integer-order differential equation (ODE)

$$x^{(1)}(t) = G_{\text{int}}\left(x(t), u(t), t\right)$$

Fractional-order differential equation (FDE) generalization

$$x^{(1)}(t) + k\left({}_0D_t^{\alpha} x\right)(t) = G\left(x(t), u(t), t\right) \qquad x(0) = x_0$$

FDE assumptions

- k = const
- Highest derivative is of integer order
- Non-zero initial conditions

EDFA ODE Model

$$x_1^{(1)}(t) = -x_1(t) + x_2(t)\xi\left(1 - e^{(B_p x_1 - A_p)}\right) + \eta\left(1 - e^{(B_s x_1 - A_s)}\right) + g_{ASE}(x_1, x_2, t)$$

EDFA FDE Model

$$x_1^{(1)}(t) + k\left({}_0D_t^{\alpha} x_1\right)(t) = -x_1(t) + x_2(t)\xi\left(1 - e^{(B_p x_1 - A_p)}\right) + \eta\left(1 - e^{(B_s x_1 - A_s)}\right)$$

$$x_2^{(1)}(t) = u(t)$$

Fractional Derivatives In Modeling Of EDFAs

There is no general approach of solving nonlinear fractional differential equations; we use the following formula*

$$x^{(\alpha)} = \frac{x(t)}{\Gamma(1-\alpha)} t^{-\alpha} - \frac{1}{\Gamma(2-\alpha)\Gamma(\alpha-1)} \sum_{p=2}^{N} \frac{\Gamma(p-1+\alpha)}{(p-1)!} \left\{ \frac{x(t)}{t^{\alpha}} + \frac{\tilde{V}_p(t)}{t^{p-1+\alpha}} \right\}$$

Or, as in this particular case, modified series development, used in the simulations

$$f^{(\alpha)}(t) \approx \frac{1}{\Gamma(2-\alpha)} \left\{ f^{(1)}(t) \left[1 + \sum_{p=1}^{N} \frac{\Gamma(p-1+\alpha)}{\Gamma(\alpha-1)\, p!} \right] t^{1-\alpha} - \left[\frac{(\alpha-1)}{t^{\alpha}} f(t) + \sum_{p=2}^{N} \frac{\Gamma(p-1+\alpha)}{\Gamma(\alpha-1)(p-1)!} \left(\frac{f(t)}{t^{\alpha}} + \frac{\tilde{V}_p}{t^{p-1+\alpha}} \right) \right] \right\}$$

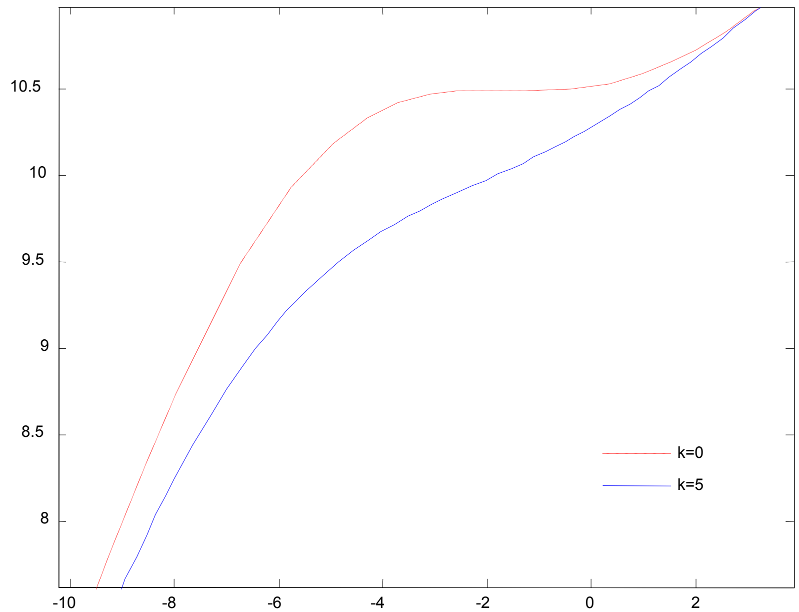
$$\tilde{V}_p(t) = -(p-1) t^{p-2} f(t), \qquad \tilde{V}_p(0) = 0, \qquad p = 2,3,\ldots N$$

- V_p's are new variables needed to take FDE to an ODE
- Our numerical experiments yielded that seven of these variables is an optimal case for our application
- This is first applied to EDFA model (without ASE) and reported at SIAM 2007

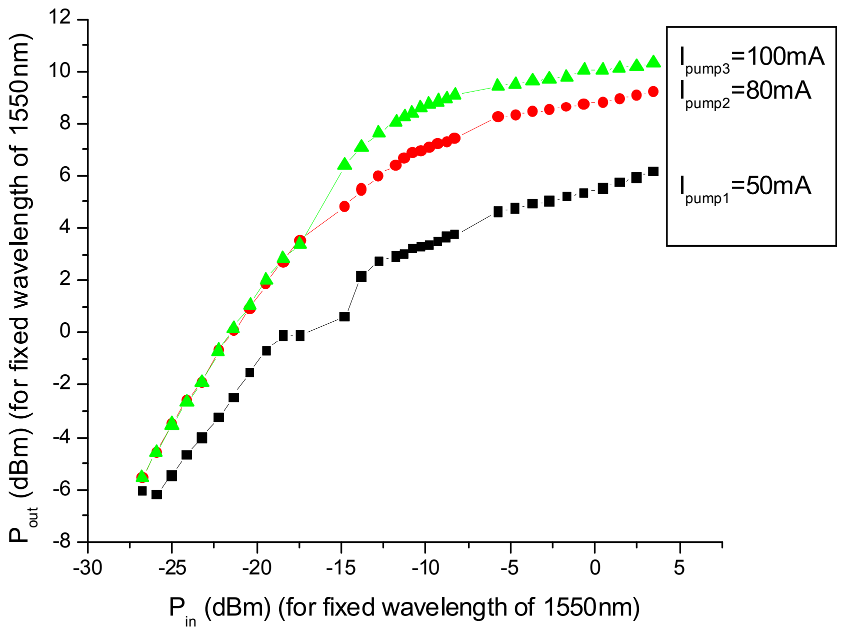
*T.M. Atanackovic , B. Stankovic, An expansion formula for fractional derivatives and its application, *Fractional Calculus and Applied Analysis*, Volume 7, Number 3, 2004.

Main Result

- Assumed one light source (d=1) entering at 80mW
- EDFA ODE model (k=1) highly nonlinear; EDFA FDE model linearizes the model, more similar to experimental results



FDE vs ODE EDFA models for one input signal



Experimental EDFA results with one input signal

Conclusions

- This is an attempt to research physical layer of optical networks from systems' perspective
- Already improved the gain of EDFAs applying nonlinear control techniques like feedback linearization and specific optimal control on ODE model
- FDE model applied here to further improve the model of EDFAs
- Optimal FDE model applied to optimize the model described here*

* Jeličić Z, Petrovački N: Optimality Conditions And A Solution Scheme For Fractional Optimal Control Problems, in *Journal of Structural and Multidisciplinary Optimization*, Springer, New York-Berlin, first published online at www.springerlink.com on August 22nd 2008

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