

# Mueller Matrix based Modeling of Nonlinear Polarization Rotation in a Tensile-Strained Bulk SOA

Michael J. Connelly and Li-Qiang Guo

Optical Communications Research Group,  
Department of Electronic and Computer Engineering,  
University of Limerick,  
Ireland.



# Outline

1. Introduction
2. Non-linear polarization rotation and Stokes parameters
3. SOA Mueller matrix
4. Experiment and simulations
5. Conclusions

# Introduction

Nonlinear polarization rotation (NPR) in SOAs has attracted much interest for use in all-optical signal processing applications such as wavelength conversion and optical logic\*.

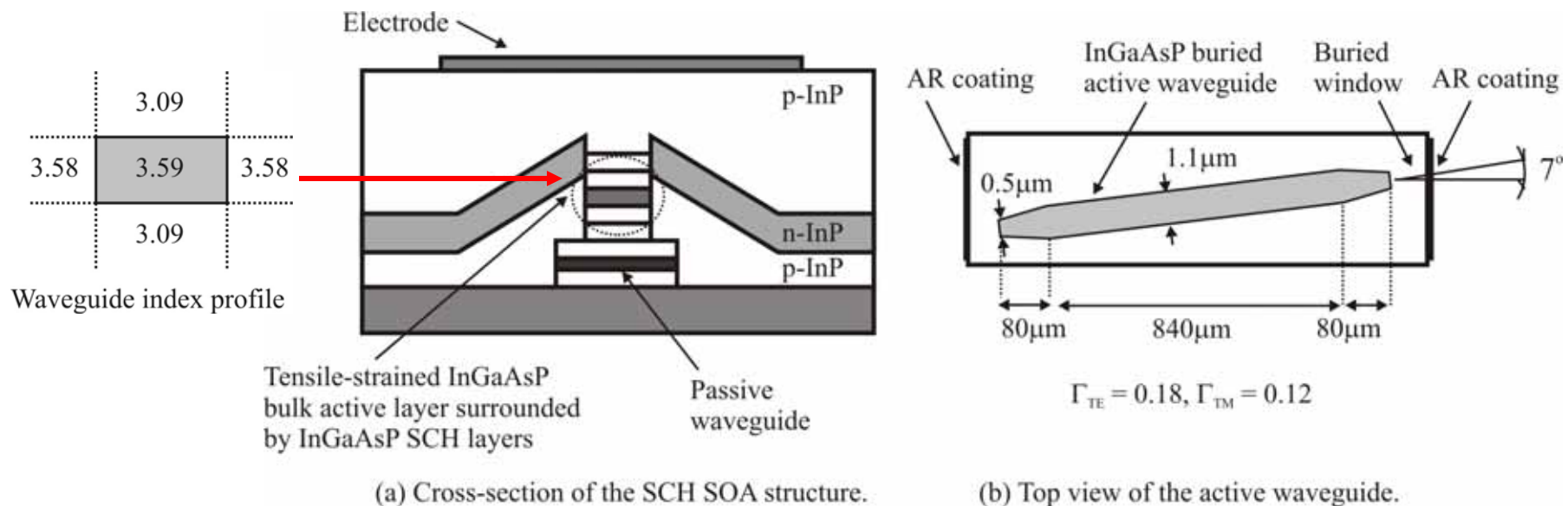
There is a need for detailed models that can predict NPR, which pay close attention to the underlying physics of a particular SOA structure and material.

We model NPR induced on a probe signal due to a co-propagating pump signal in a tensile-strained bulk SOA, using a Mueller matrix approach.

\* L.Q. Guo and M.J. Connelly, *J. Lightwave Technol.*, 2005.

# Tensile-Strained SOA

Tensile-strained bulk SOAs have attracted much interest due to its relative ease of fabrication and commercial devices are now available.



Device geometry and some material parameters supplied by *Amphotonics\** (Kamelian).

\* C. Michie et al., *J. Lightwave Technol.*, 2006.

# Steady-state model

We have previously developed a wideband steady-state model\* that can be used to predict the tensile-strained SOA behaviour without the use of any adjustable parameters.

- uses a detailed model for the material band-structure.

For a given bias current and input signal powers the model can be used to predict the spatial distribution of the carrier density.

\* M.J. Connelly, *IEEE J. Quantum Electron.*, 2007

# Non-Linear Polarization Rotation

NPR in an SOA is caused by the *carrier density dependence* of the TE and TM mode active region refractive index. The polarization state of a weak probe signal can be changed by the presence of a strong pump.

The NPR of the probe can be modeled using Mueller matrices\*. If the Stokes vector  $S_i$  of the input probe signal is known, then the probe output Stokes vector is

$$S = MS_i$$

where  $M$  is the SOA Mueller matrix.

This is particularly useful because it is easy to measure Stokes vectors using a polarisation analyser.

\* L.Q. Guo and M.J. Connelly, *J. Lightwave Technol.*, 2007.

# Stokes Parameters

Any state of polarized light can be described by 4 measurable quantities (Stokes parameters or vector)  $(S_0, S_1, S_2, S_3)^T$ .

A plane wave propagating in the z direction can be considered to be the sum of two orthogonal plane waves.

$$E_x(t) = E_{0x}(t) \cos[\omega t + \delta_x(t)] \quad \text{and} \quad E_y(t) = E_{0y}(t) \cos[\omega t + \delta_y(t)]$$

It can be shown that

$$(E_{0x}^2 + E_{0y}^2)^2 - (E_{0x}^2 - E_{0y}^2)^2 - (2E_{0x}E_{0y} \cos \delta)^2 = (2E_{0x}E_{0y} \sin \delta)^2$$

$$\text{or } S_0^2 = S_1^2 + S_2^2 + S_3^2$$

$$\delta = \delta_y - \delta_x$$

$S_0$  is the total light intensity.

$S_1$  describes the amount of linear horizontal or vertical polarization.

$S_2$  describes the amount of linear  $+45^\circ$  or  $-45^\circ$  polarization.

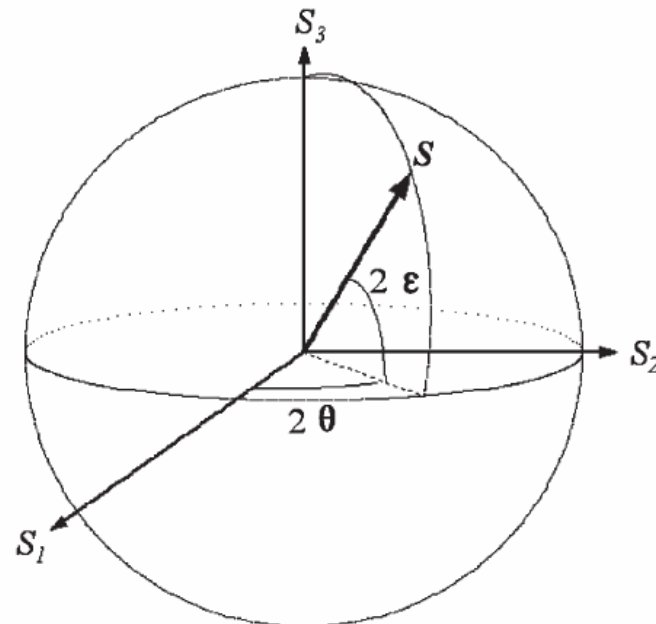
$S_3$  describes the amount of right or left circular polarization.

The Stokes parameters are expressed in terms of **intensities** and are easily measured using a **polarization analyzer**.

They can be displayed on the **Poincare sphere**.

$\theta$  polarization azimuth.

$\mathcal{E}$  ellipticity angle.





# SOA Mueller Matrix

The SOA Mueller matrix is composed of a *diattenuator* (as the TE and TM gains are not equal) followed by a *retarder* (phase shifter)\*.

$$M = M_{dia} M_{ret}$$

$$M_{dia} = \frac{1}{2} \begin{pmatrix} G_{TE} + G_{TM} & G_{TE} - G_{TM} & 0 & 0 \\ G_{TE} - G_{TM} & G_{TE} + G_{TM} & 0 & 0 \\ 0 & 0 & 2\sqrt{G_{TE}G_{TM}} & 0 \\ 0 & 0 & 0 & 2\sqrt{G_{TE}G_{TM}} \end{pmatrix}$$

$$M_{ret} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \Delta\phi & \sin \Delta\phi \\ 0 & 0 & -\sin \Delta\phi & \cos \Delta\phi \end{pmatrix}$$

\* L.Q. Guo and M.J. Connelly, *J. Lightwave Technol.*, 2007.

# TE-TM Phase Shift

The TE-TM phase shift is given by

$$\Delta\phi = \frac{2\pi}{\lambda_{probe}} \int_0^L [N_{eff,TE}(z) - N_{eff,TM}(z)] dz$$

$N_{eff}(z)$  is the spatial and polarisation dependent *effective index*.

We can write

$$N_{eff,TE/TM}(z) = N_{eff,TE/TM,0}(z) + \frac{dN_{eff,TE/TM}(z)}{dN_1} \Delta N_{1,TE/TM}(n(z))$$

← **Active region  
refractive index.**

$N_{eff,TE/TM,0}(z)$  is the effective index with no injected carriers.  
and was determined using Marcatili's method for rectangular dielectric waveguides\*.

\*S.H. Chuang, *Physics of Optoelectronic Devices*, Wiley, 1995.  
NUSOD-08

# Carrier Density induced Refractive Index Change

The change in the active region refractive index due to the injected carrier density is due to two main effects, *bandfilling* and *free-carrier absorption*.

$$\Delta N_{1,TE/TM}(n) = \Delta N_{bf,TE/TM}(n) + \Delta N_{fca}(n)$$

The change due to bandfilling is given by

$$\Delta N_{bf,TE/TM}(E) = \frac{2c\hbar}{e^2} \text{PV} \int_0^\infty \frac{\alpha_{TE/TM}(E',n) - \alpha_{TE/TM}(E',0)}{E'^2 - E^2} dE'$$

where  $\alpha_{TE/TM}(E,n)$  is the material absorption.

The index change due to FCA is usually given by

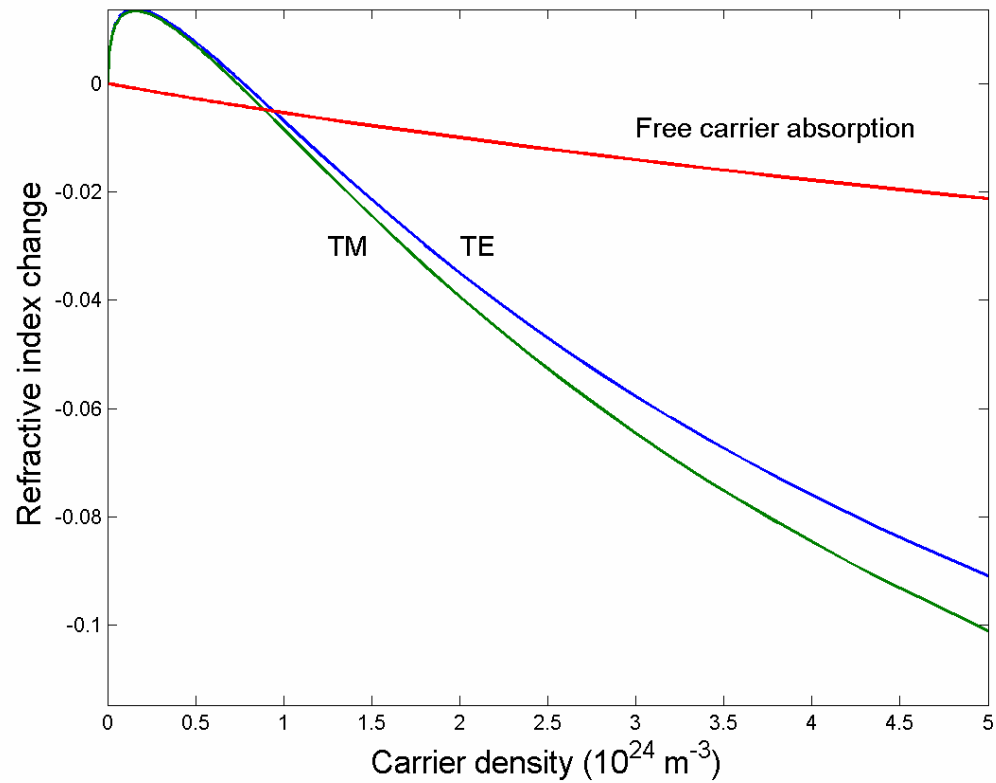
$$\Delta N_{fca}(n) = -\frac{e^2 \hbar^2}{2N_1 \epsilon_0 E^2} \left( \frac{n}{m_c} + \frac{p}{m_v} \right)$$

This is not the case for a tensile-strained material, where the effective masses are not constant.

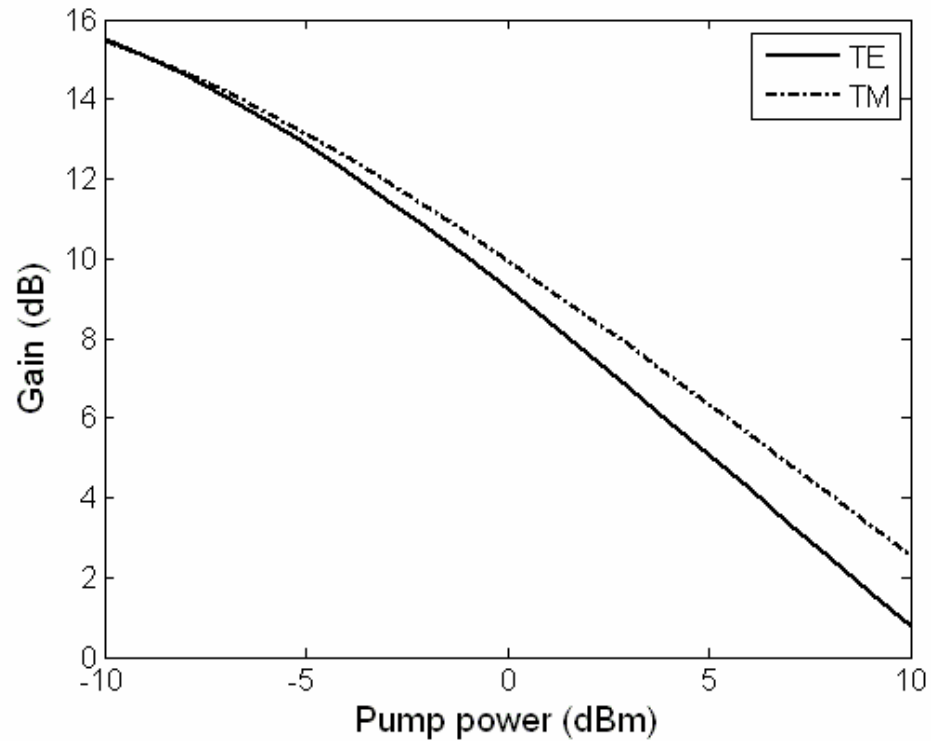
The conduction band contribution is given by

$$\Delta N_{fca,c}(n) = -\frac{e^2 \hbar^2}{2N_1 \epsilon_0 E^2} \int_0^\infty \frac{k^2}{\pi^2 m_c(k)} \left[ 1 + \exp\left(\frac{E_c(k) - E_{fc}}{kT}\right) \right]^{-1} dk$$

Slightly more complicated expressions are used for the contributions of the heavy- and light-hole valence bands.

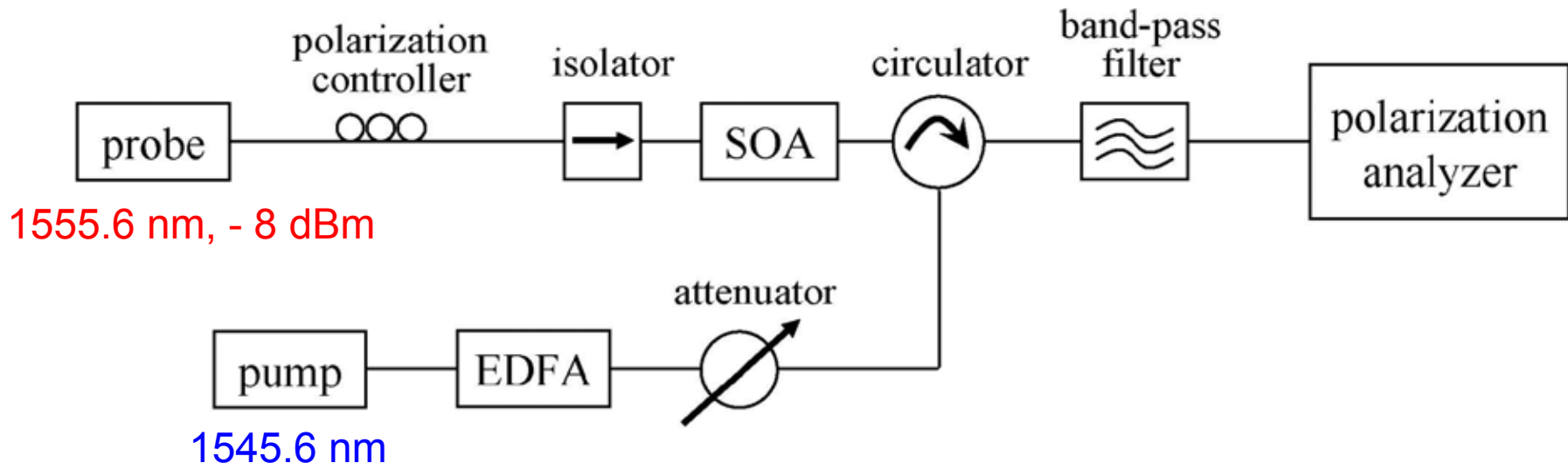


**Active region refractive index change v.s. carrier density.**



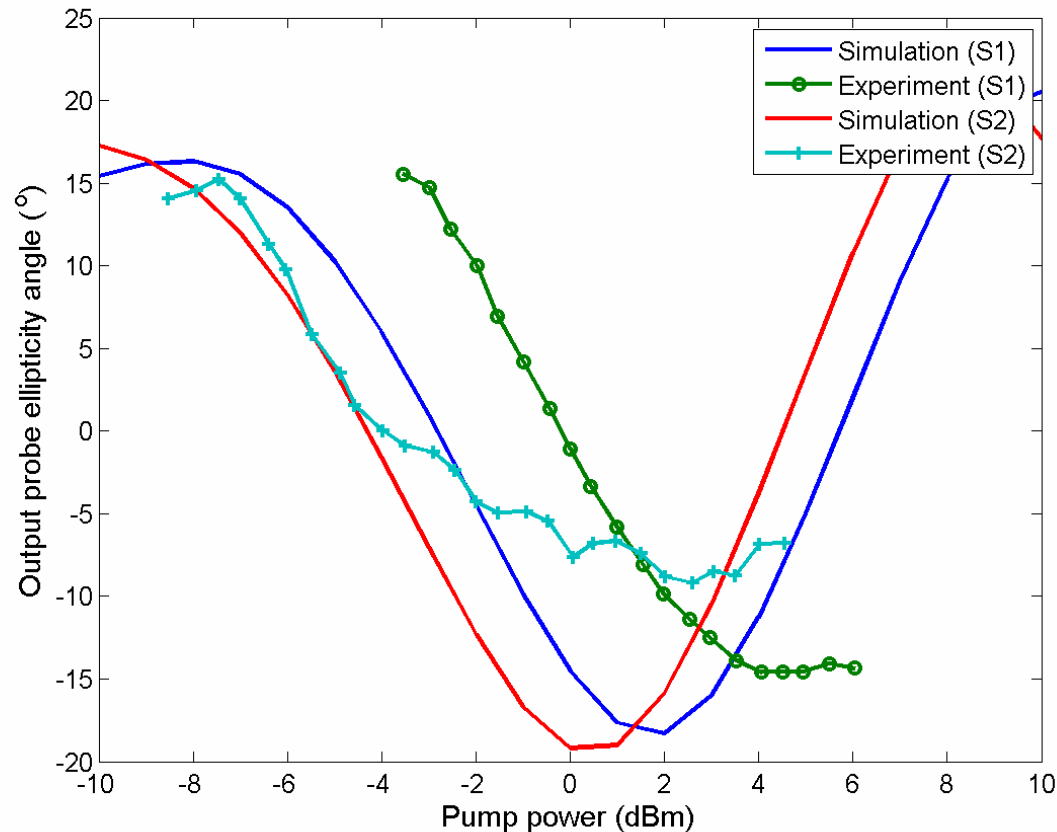
**Simulated probe TE and TM gain versus pump power.  
Probe input power = -8 dBm.**

# Experiment and Simulations



**Counter-propagating pump and probe.**

Stokes vector of the output probe measured using a polarization analyzer.



$$S1 = [1, 0.73, -0.46, 0.13]$$

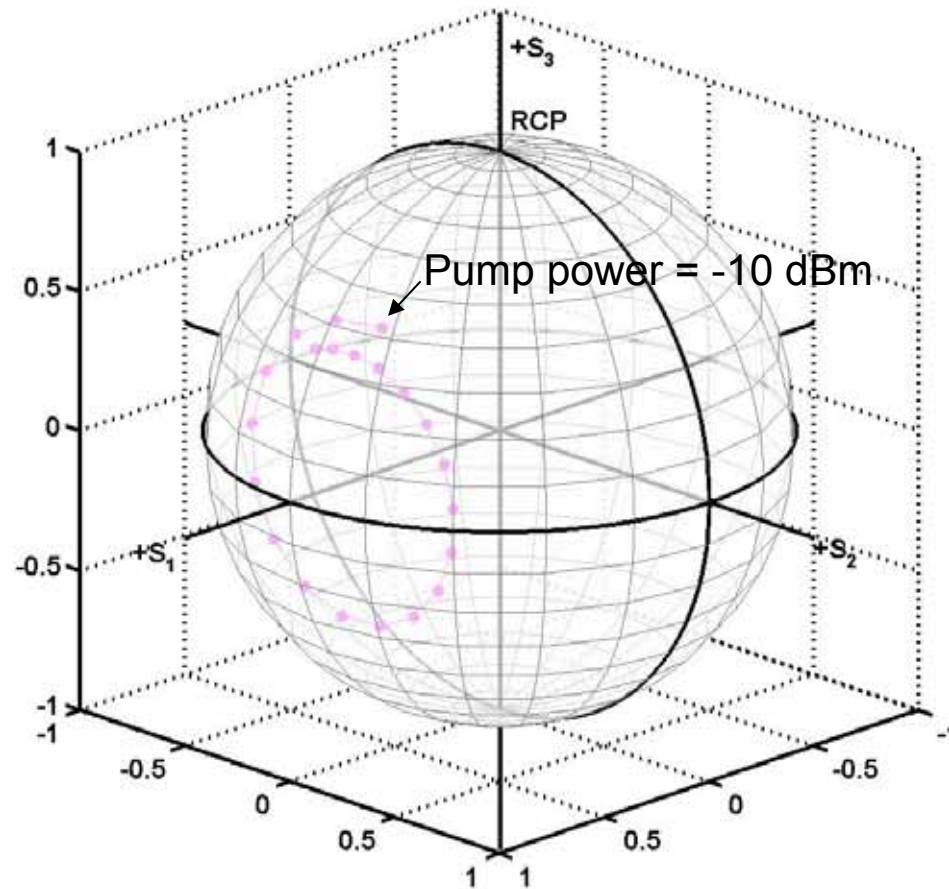
$$S2 = [1, 0.75, -0.4, 0.37]$$

Ellipticity angle

$$\varepsilon = 0.5 \tan^{-1} \left( \frac{S_3}{\sqrt{S_1^2 + S_2^2}} \right)$$

For pump powers between  $-7$  and  $7$  dBm, the rolloff of the ellipticity-angle trajectories as predicted by the model are in good agreement with the experiment.





**Evolution of probe polarization state shown on the Poincare sphere.**

Possible discrepancies between experiment and simulation include.

- Uncertainty in pump and probe input powers and the actual proportion coupled to the waveguide TE and TM modes.
- Sensitivity of NPR experiments.
- Inaccuracies in estimation of TE and TM effective indices.

# Conclusions

Developed a Mueller matrix technique for predicting NPR in an SOA, which uses no adjustable parameters.

Reasonably good agreement between experiment and simulations.

Future work will examine NPR for a wider range of input probe polarization conditions.