Multi-population rate equation simulation of quantum dot lasers with feedback

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OUTLINE

- Introduction and motivations
- Multi-population rate equation for carrier dynamics in quantum dots is presented
- Inclusion of weak external optical feedback
- Simulation results for Single Longitudinal Mode (SLM) Laser are presented
- Comparison with an equivalent QW case
- Conclusions

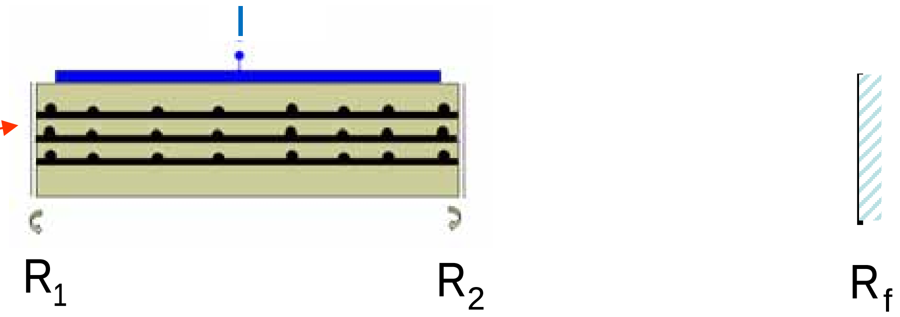
Introduction and motivations

QD semiconductor lasers and weak external feedback:

QD active region:

N layers of **InAs/GaAs** QDs in a **DWELL** structure

Emission from **GS around 1.3 μm**



- It was predicted that QD lasers can be less sensitive to optical feedback than Qwell or bulk lasers thanks to the very low α-parameter and the high gain compression
- Several experiments and models have however shown that the α-parameter can also be high and very dependent on working conditions
- Needs of models to study and understand the effects of external feedback in QD lasers

The Model

Typical models analyzing feedback in QD lasers

- do not include inhomogeneous distribution of QD size
- do not include the presence of the Excited States
- use a CONSTANT α parameter independent on working conditions

Ex. D.O'Brien et al."Sensitivity of QD lasers to optical feedback", ***Opt.Letters,*** **May 2004**

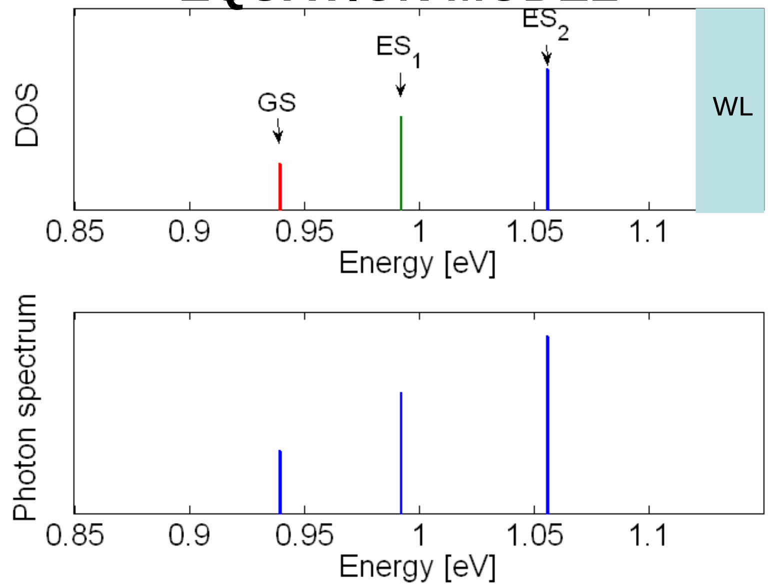
Our objective is to develop a model that:

- includes the inhomogeneous distribution of the QD size
- includes the ES, always present with QDs at 1.3 μm
- uses only physical parameters and not equivalent parameters exctracted from small signal measurements (i.e: α-parameter, differential gain, ...)

↳ The model is used to analyze the SLM laser response versus time with weak external optical feedback

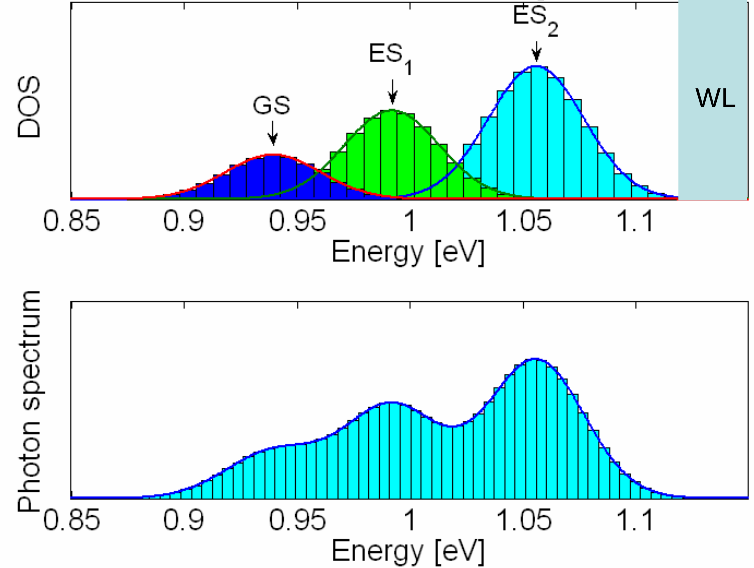
Existing Models

SIMPLE RATE EQUATION MODEL



- A single rate equation for each confined QD state and for the corresponding emissions.
- **No information on the emission spectrum**, only on total optical power emitted from GS and ES
- **Very low computational cost**

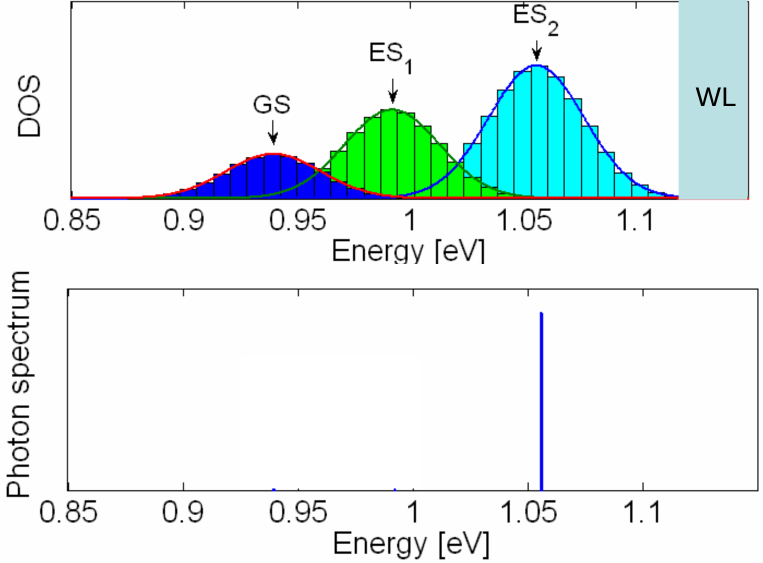
MULTI POPULATION RATE EQUATION MODEL (MPRE)



- QD population is subdivided in many subpopulations to represent QD size dispersion
- The photon population is represented with a spectrally resolved model
- **High accuracy** in modeling of QD based devices
- **High computational cost**

Existing Models

MULTI POPULATION RATE EQUATION MODEL (MPRE)



- ☐ QD population is subdivided in many subpopulations to represent QD size dispersion
- ☐ We consider a single longitudinal mode laser
- ☐ **High accuracy** in modeling of QD based devices

Multi-population rate equation for carriers

A system of coupled rate equation

- one RE for carriers in the WL,
- several rate equations for carriers in the ES and GS to account for the inhomogeneous broadening

$$\text{WL:}\quad \frac{dN_{WL}}{dt} = \frac{I}{e} + \sum_n N_{ESn}\frac{1}{\tau_{ESn}^{WL}} - \sum_n N_{WL}\frac{1}{\tau_{cn}} - N_{WL}\frac{1}{\tau_{qr}},\ n = 1,2\ldots N$$

$$\text{ES}_n\text{:}\quad \frac{dN_{ESn}}{dt} = N_{WL}\frac{1}{\tau_{cn}} + N_{GSn}\rho_{ESn}\frac{1}{\tau_{GSn}^{ESn}} - N_{ESn}\frac{1}{\tau_{ESn}^{WL}} - N_{ESn}\rho_{GSn}\frac{1}{\tau_{dn}} - N_{ESn}(1-\rho_{ESn})\frac{1}{\tau_{sp}^{ESn}} - v_g\Gamma g_m^{ESn} S_m - R_{Aug}^{ESn},\ n = 1,2\ldots N$$

$$\text{GS}_n\text{:}\quad \frac{dN_{GSn}}{dt} = N_{ESn}\rho_{GSn}\frac{1}{\tau_{dn}} - N_{GSn}\rho_{ESn}\frac{1}{\tau_{GSn}^{ESn}} - N_{GSn}(1-\rho_{GSn})\frac{1}{\tau_{sp}^{GSn}} - v_g\Gamma g_m^{GSn} S_m - R_{Aug}^{GSn},\ n = 1,2\ldots N$$

photons in the lasing mode

Gain variation at the lasing wavelength:

$$g(E) = C_g\ N_D \sum_n \sum_k \mu_k \frac{|P_k^\sigma|^2}{E_k} \left(2f_k(E_{k_n}) - 1\right) G_n\ B_k(E - E_{k_n})$$

sum over QD sub-groups

sum over QD states (GS, ES,...)

Carrier variation and refractive index change

Refractive index change at lasing wavelength*:

$$\Delta n_{eff\ tot}(E_j) = \Delta n_{plasma}(E_j) + \Delta n_{QD}(E_j)$$

- *Refractive index change due to carriers in WL and SCH (free carrier or plasma contribution)*

$$\Delta n_{\mathrm{plasma}}(E_j) = \Gamma_{\mathrm{SCH}} K_{\mathrm{plasma}} \frac{\Delta N_{\mathrm{s}}}{E_j^2} + \Gamma_{\mathrm{WL}} K_{\mathrm{plasma}} \frac{\Delta N_{\mathrm{q}}}{E_j^2}$$

- *Refractive index change due to carriers confined in the QDs: Kramer-Kronig term*

$$\Delta n_{\mathrm{QD}}(E_j) = \Gamma \frac{\hbar\, c}{2\, E}\, C_{\mathrm{g}} N_{\mathrm{D}} \sum_k \sum_m \mu_k \frac{|P_k^\sigma|^2}{E_k}\, (2P_{k_m} - 1)\, G_m\, D_{\mathrm{cv}}(E_j - E_{k_m})$$

sum over QD states

sum over QD sub-groups

Homogeneous broadening function of refractive index

$$D_{\mathrm{cv}}(E_j - E_{k_m}) = \frac{(E_j - E_{k_m})/\pi}{(E_j - E_{k_m})^2 + (\hbar\Gamma_{\mathrm{hom}})^2}$$

* *M. Gioannini, I. Montrosset, "Numerical Analysis of the frequency chirp in QD semiconductor lasers", IEEE J. Quantum Electron. October 2007*

Coupling with the Electric Field

We define the electric field at the lasing wavelength of the SLM:

$$E(t) = \tilde{E}(t) \cdot e^{j\omega_r t}$$

slowly varying component

reference pulsation (cold cavity resonance)

Delayed differential equation coupled with the MPRE for carriers:

$$\frac{d\tilde{E}(t)}{dt} = -\frac{\tilde{E}(t)}{2\tau_p} + \frac{v_g g(E,t)\tilde{E}(t)}{2} + j2\pi\delta f(t)\tilde{E}(t) + k_{feedback}\tilde{E}(t-\tau_{ext})$$

photon loss

stimulated emission rate

frequency chirp due to carrier fluctuations:

$$\delta f(t) = -\frac{E_{\mathrm{mode}}}{h n_r}\Delta n_{eff\ tot}(t)$$

feedback strength

external cavity round trip time

Simulation results

1. QD laser under investigation: structure and input parameters

2. Two examples of laser response are presented:
   - changing the feedback strength
   - P-I characteristic for given feedback

3. To define an equivalent QW lasers the dynamic characteristics of the solitary QD laser have been extracted from small signal simulations, obtaining: α-parameter, damping factor and resonance frequency

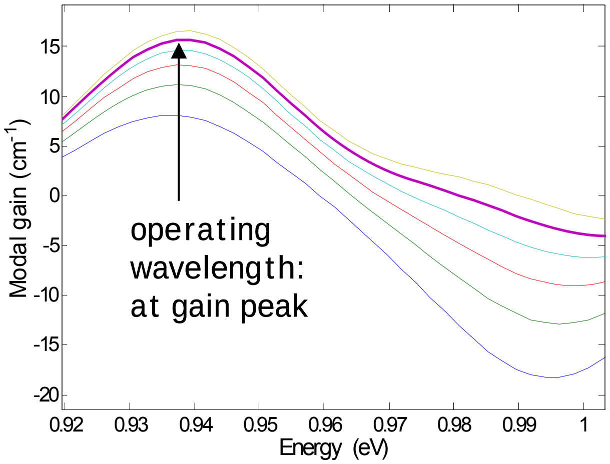
4. **Comparison with theory for Qwell or bulk lasers with external feedback has been done**

Single mode QD laser under investigation

Device and material parameters

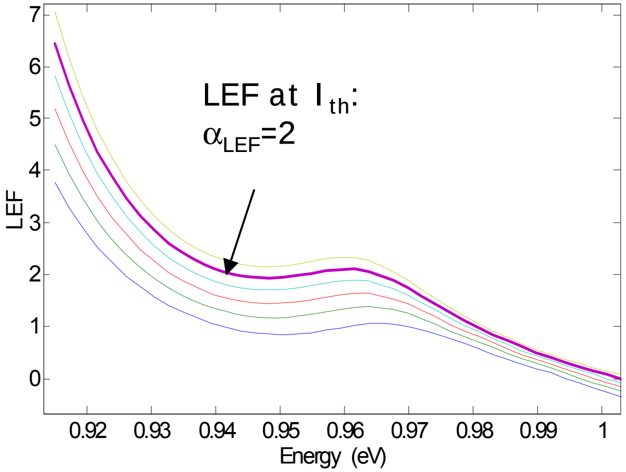
Cavity length	500 µm
Left and right reflectivity	0.7, 0.3
Number of QD layers	10
Dot density per layer	$5 \cdot 10^{10}$ cm^{-2}
Inhomogeneous broadening	30 meV
Homogeneous broadening	20 meV
Optical confinement factor	10.3 %
ES energy	0.99 eV
GS energy	0.94 eV
WL and ES energy	40 meV
Capture time from WL to ES	7 ps
Relaxation time from ES to GS	1 ps
Radiative recombination (all states)	2 ns
Auger non-radiative recombination	0.3 ns

Gain spectra



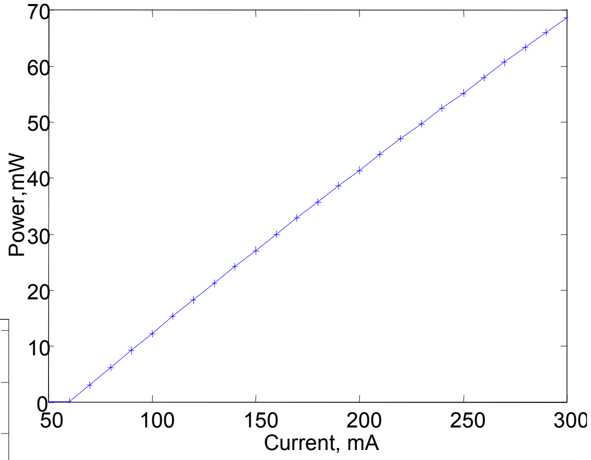
operating wavelength: at gain peak

LEF spectra



LEF at I_{th}: $\alpha_{LEF}=2$

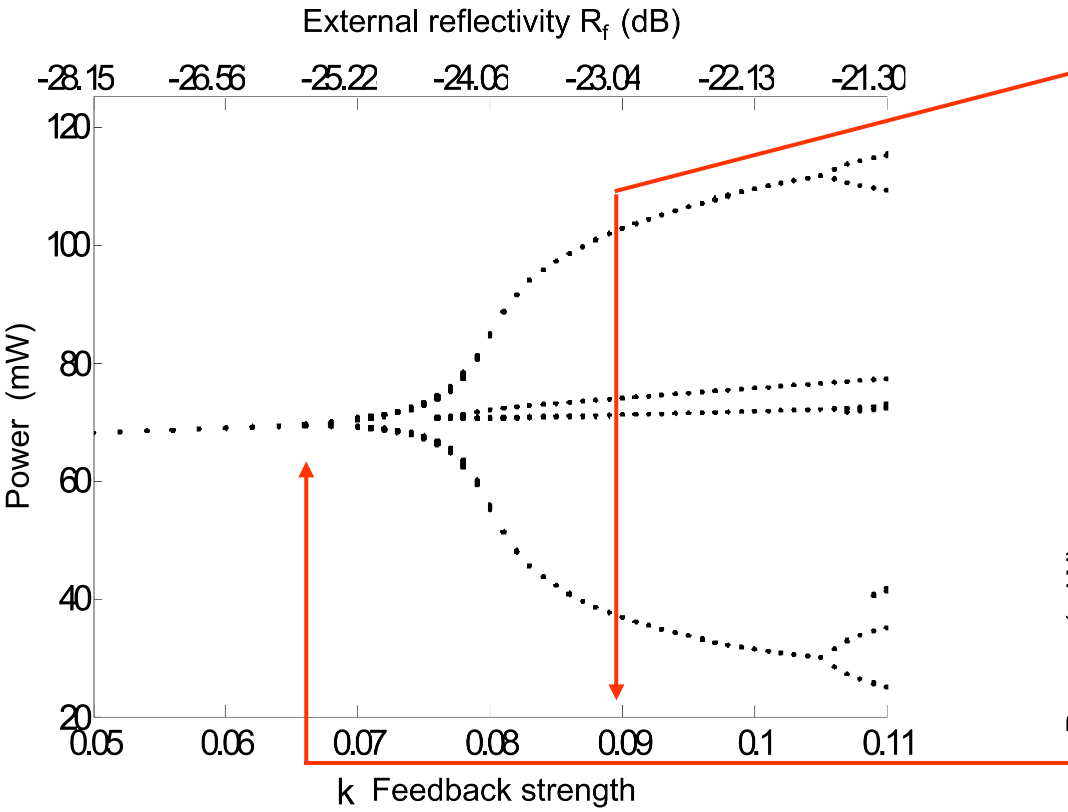
Power vs current (solitary laser)



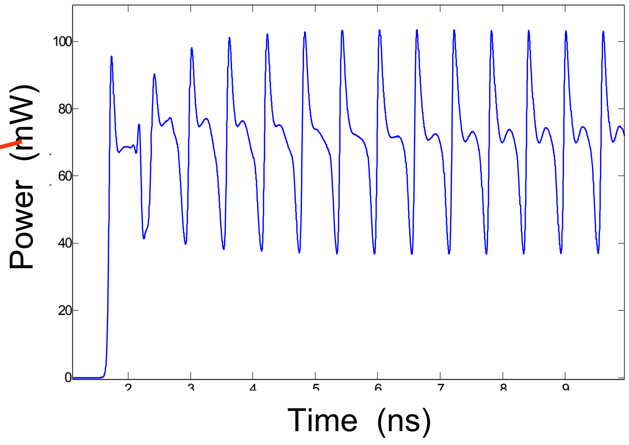
1 - Laser response varying optical feedback

We calculate the laser response for a fixed current injection I=300mA varying the external mirror reflectivity (τ_{ext}=500 ps)

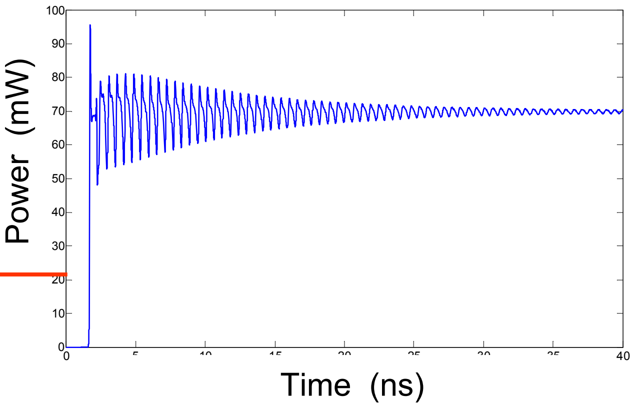
Bifurcation diagram: collection of maxima and minima of output power vs feedback strength



Periodic behavior



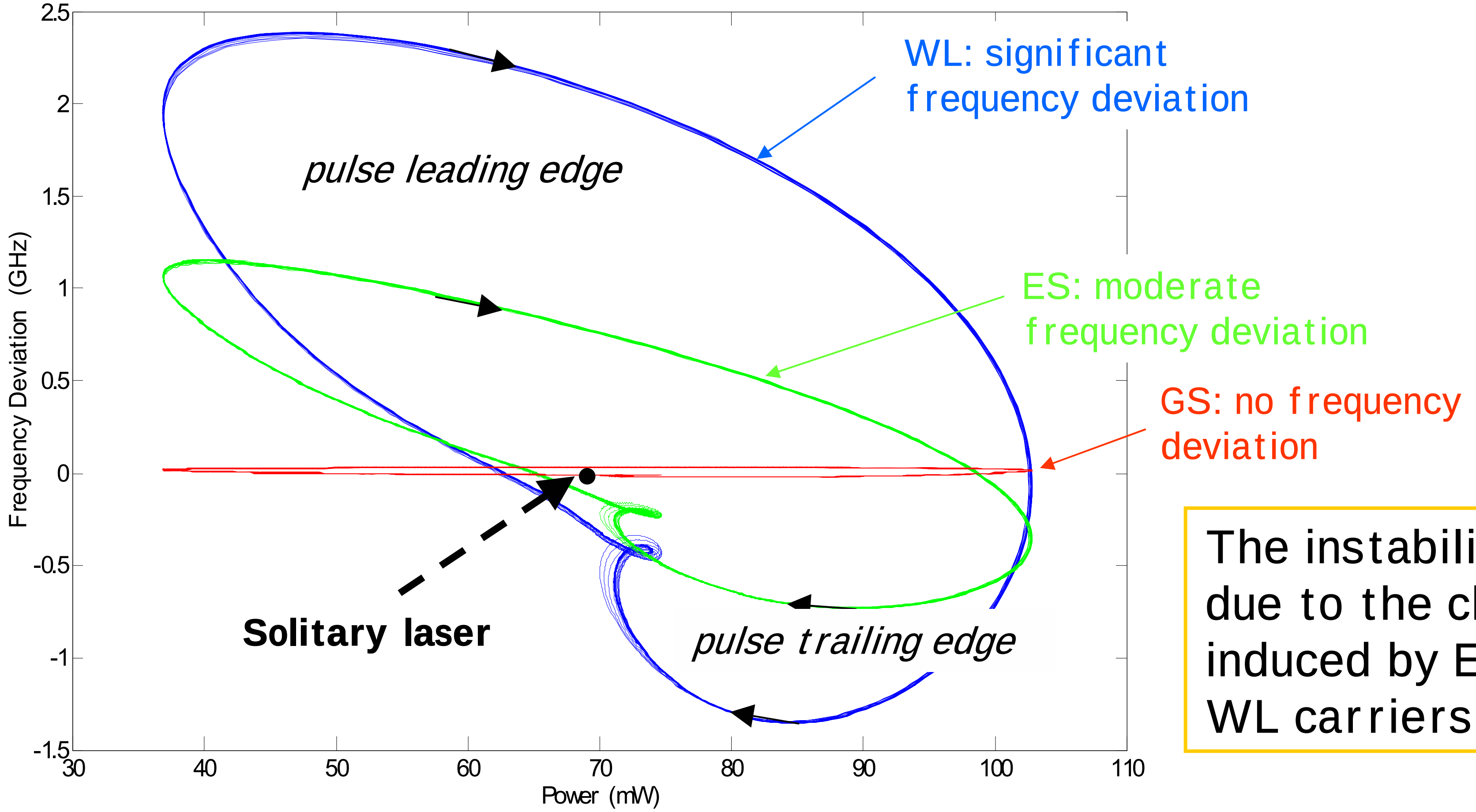
Stable points



Analysis: limit cycle with periodic behaviour

We plot in the power-frequency plane the instantaneous frequency deviation respect to the frequency of the solitary laser.

We separate the contributions due to carriers in the GS, ES, WL



WL: significant frequency deviation

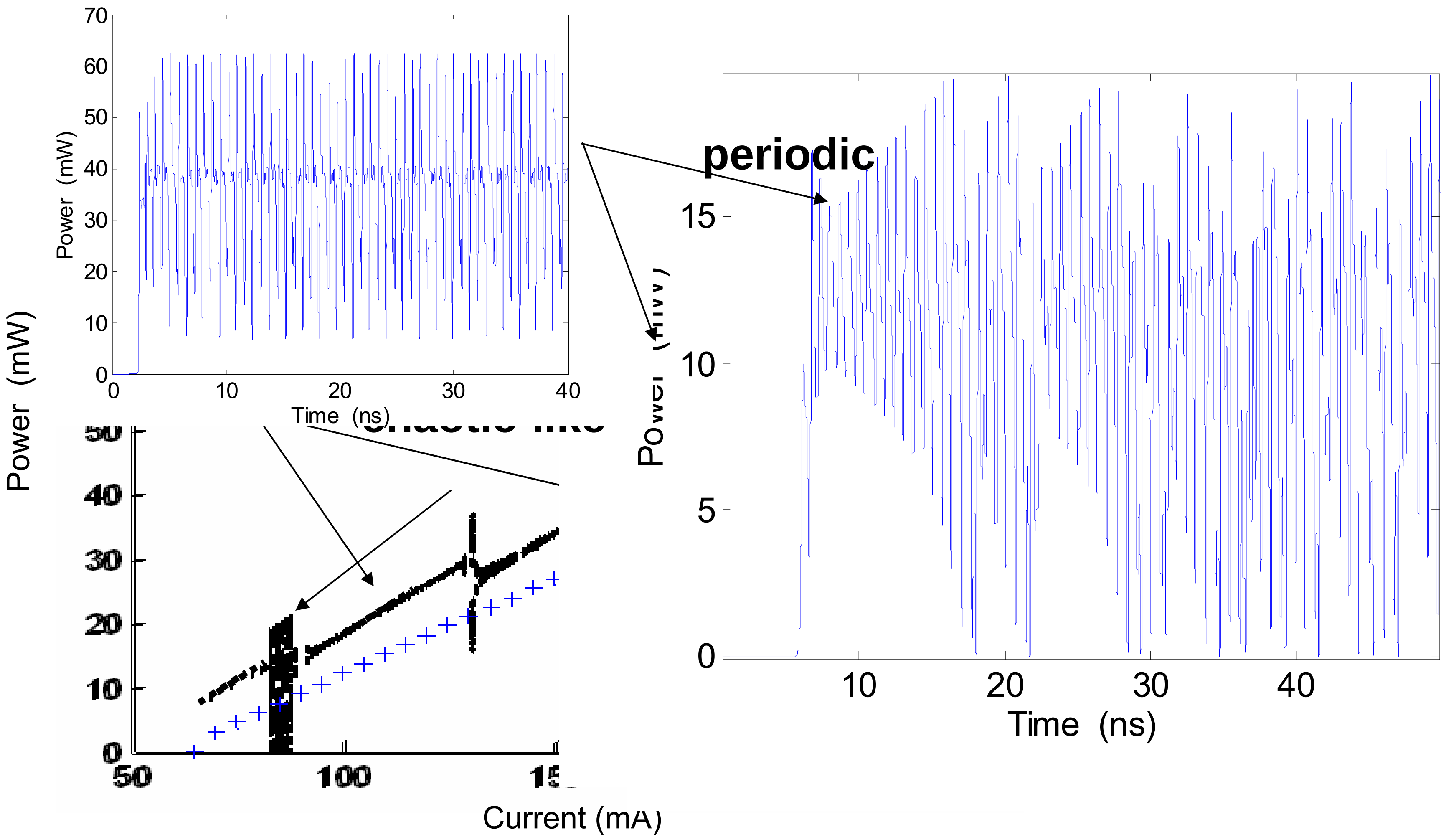
ES: moderate frequency deviation

GS: no frequency deviation

The instability is due to the chirp induced by ES and WL carriers

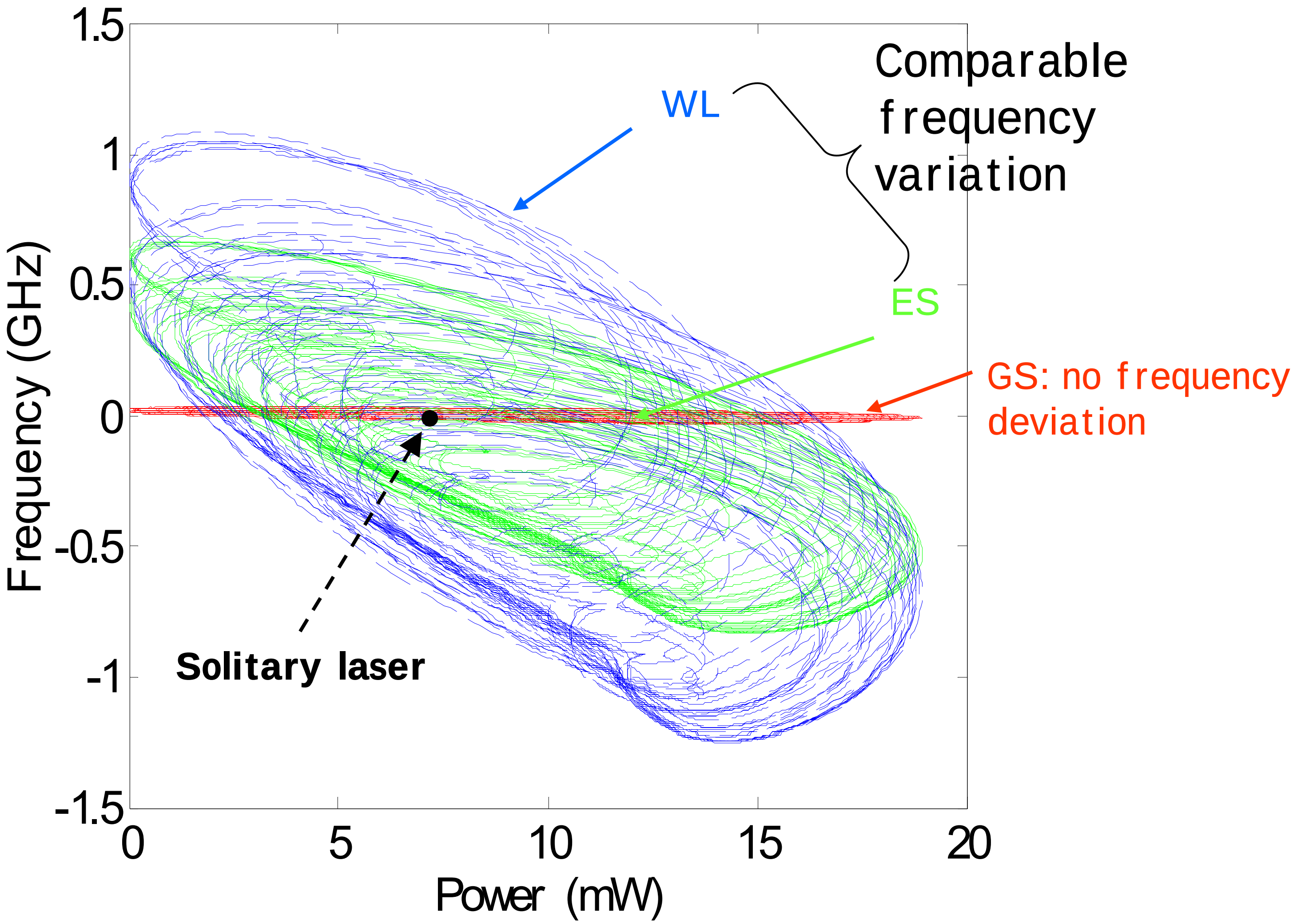
2 - Power vs current (weak optical feedback)

We calculate the laser response at fixed external reflectivity (k=0.09) varying the injection current (τ_{ext}=576 ps)



Analysis: limit cycle in “chaotic-like” point

We plot the instantaneous frequency deviation respect to the frequency of the solitary laser due to carriers in the GS, ES, WL.



Comparison with Qwell or bulk laser (1)

Stability has been largely studied for Qwell and bulk lasers using simple models with one rate equation for carriers and one for the electric field:

•the stability analysis results obtained by *J. Mørk, B. Tromborg, J. Mark, IEEE JQE, vol. 28, no. 1, January 1992*

•the simple analytic expression for stable operation condition by *J. Helms and K. Petermann, IEEE JQE, vol.26, May 1990*

These analysis show a dependence of the stability on

a)relaxation oscillation frequency,

b)damping factor and

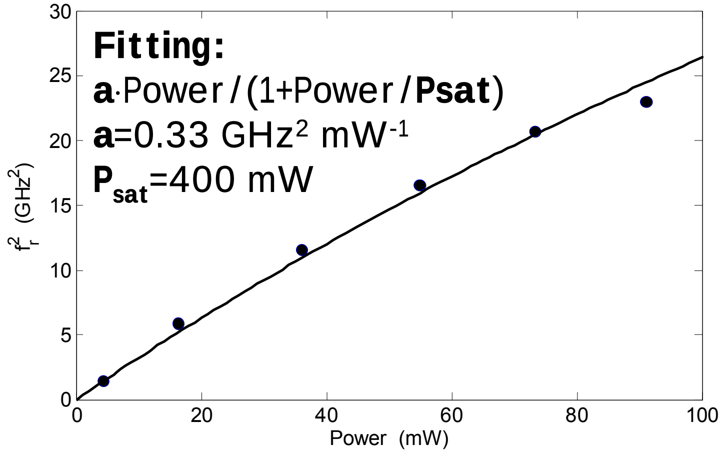
c)α-parameter of the lasers.

We define the "equivalent Qwell or bulk laser" as the solitary laser with the same output power, relaxation oscillation frequency, damping factor and α-parameter of the QD laser.

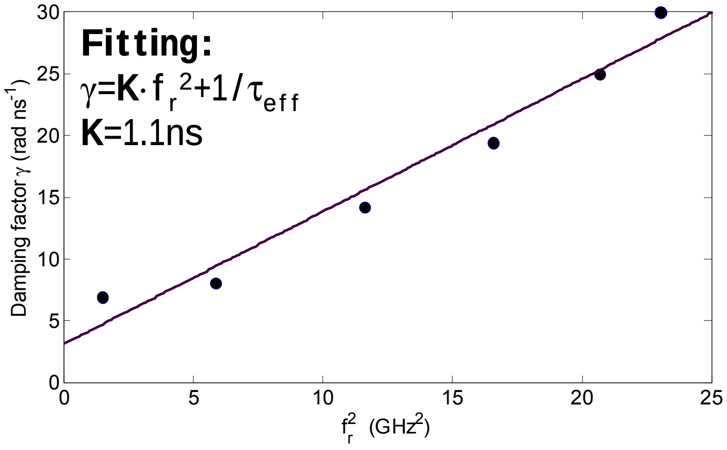
Dynamic properties of the solitary laser

From the simulated IM and FM response we can extracted *:

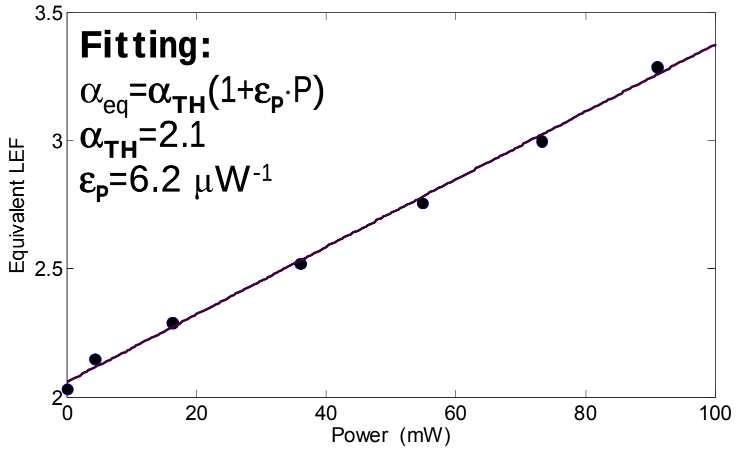
Resonance frequency



Damping factor



Equivalent α_{LEF}

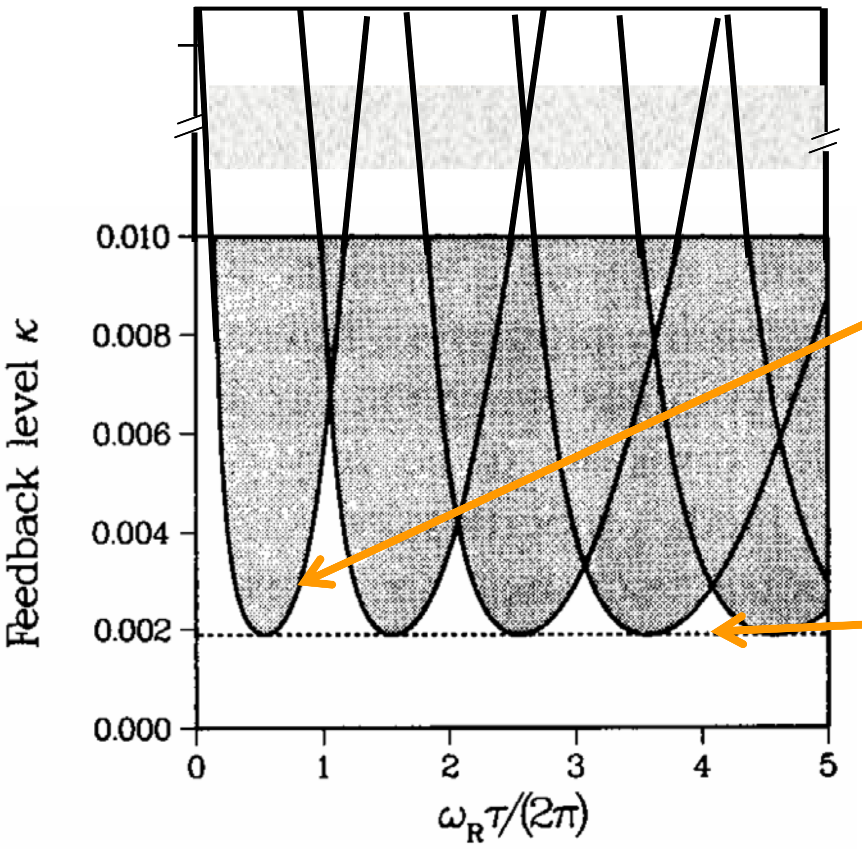


* H. SU, L. Lester, “Dynamic properties of QD DFB lasers: high speed, linewidth and chirp”, J. Phys. D: Appl. Phys., vol. 38, 2005

and we use them for the stability analysis of the equivalent Qwell laser

Comparison with Qwell or bulk laser (2)

The equivalent Qwell or bulk laser in these conditions would be **always** “unstable”



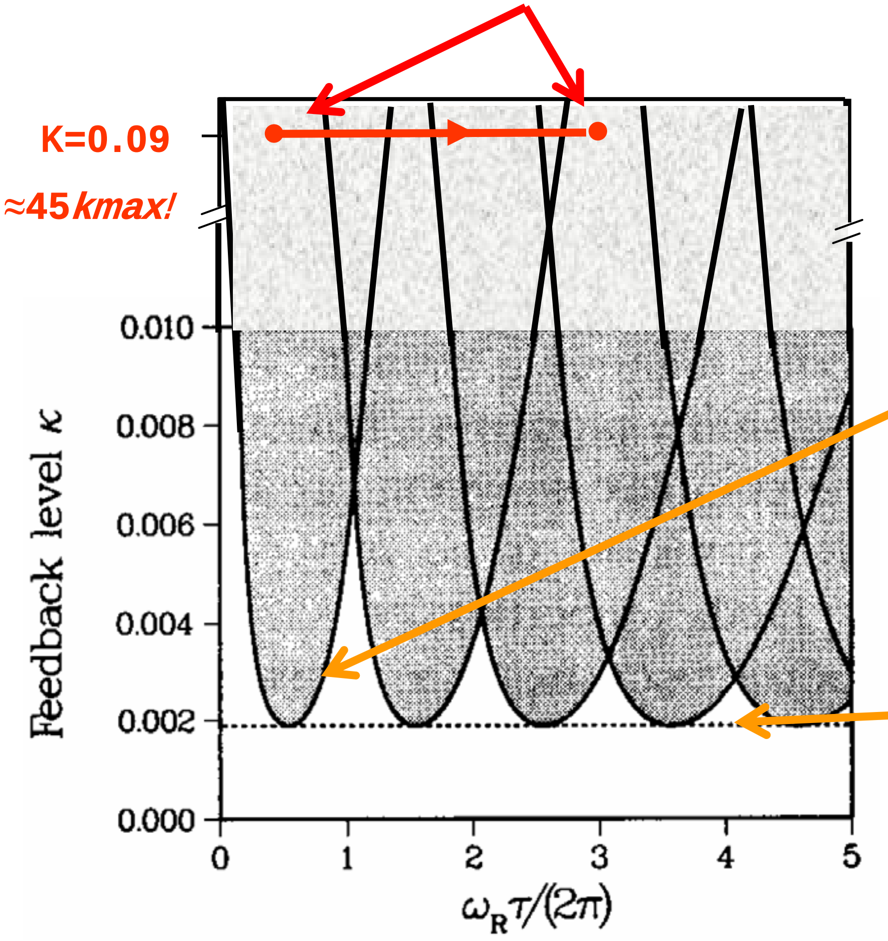
Curves of minimum feedback level to reach the stability boundary by *J. Mørk, B. Tromborg, J. Mark*

Line of maximum feedback for stable operation by *J. Helms and K. Petermann*

$$k_{\max} = \frac{\tau_{in}}{2\tau_R\sqrt{1+\alpha^2}}$$

Comparison with Qwell or bulk laser (2)

Working points for the equivalent laser with increasing current



The equivalent Qwell or bulk laser in these conditions would be **always** “unstable”

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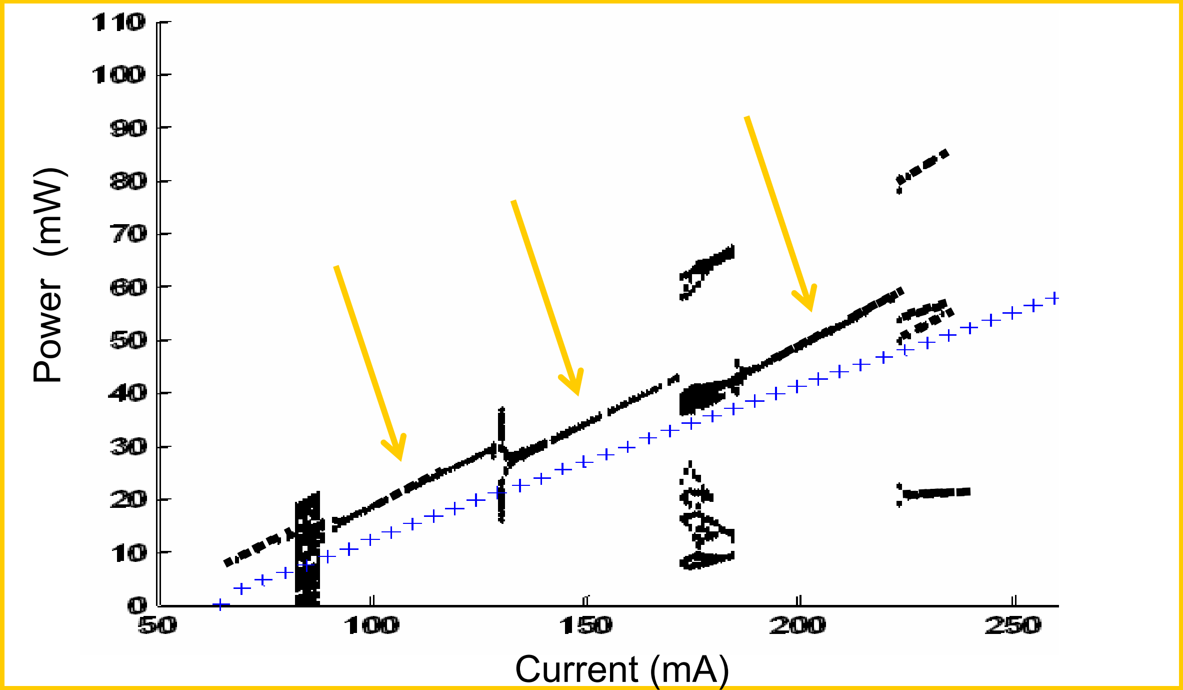
$$k_{\max} = \frac{\tau_{in}}{2\tau_R\sqrt{1+\alpha^2}}$$

Comparison with Qwell or bulk laser (2)

BUT the QD laser just analyzed is **stable** for several current ranges!!



The QD lasers are more stable than the equivalent Qwell

Conclusions and future work

Conclusions:

- We have developed a MPRE model to study QD SLM lasers with weak external optical feedback
- We have shown two examples of calculated laser response changing the feedback level and the current injection
- The results have been compared qualitatively with an equivalent Qwell or bulk laser and have shown that the QD laser is more stable.

Future work:

- Understand and compare in a more quantitative way the mechanisms leading to reduced sensitivity to feedback in QDs

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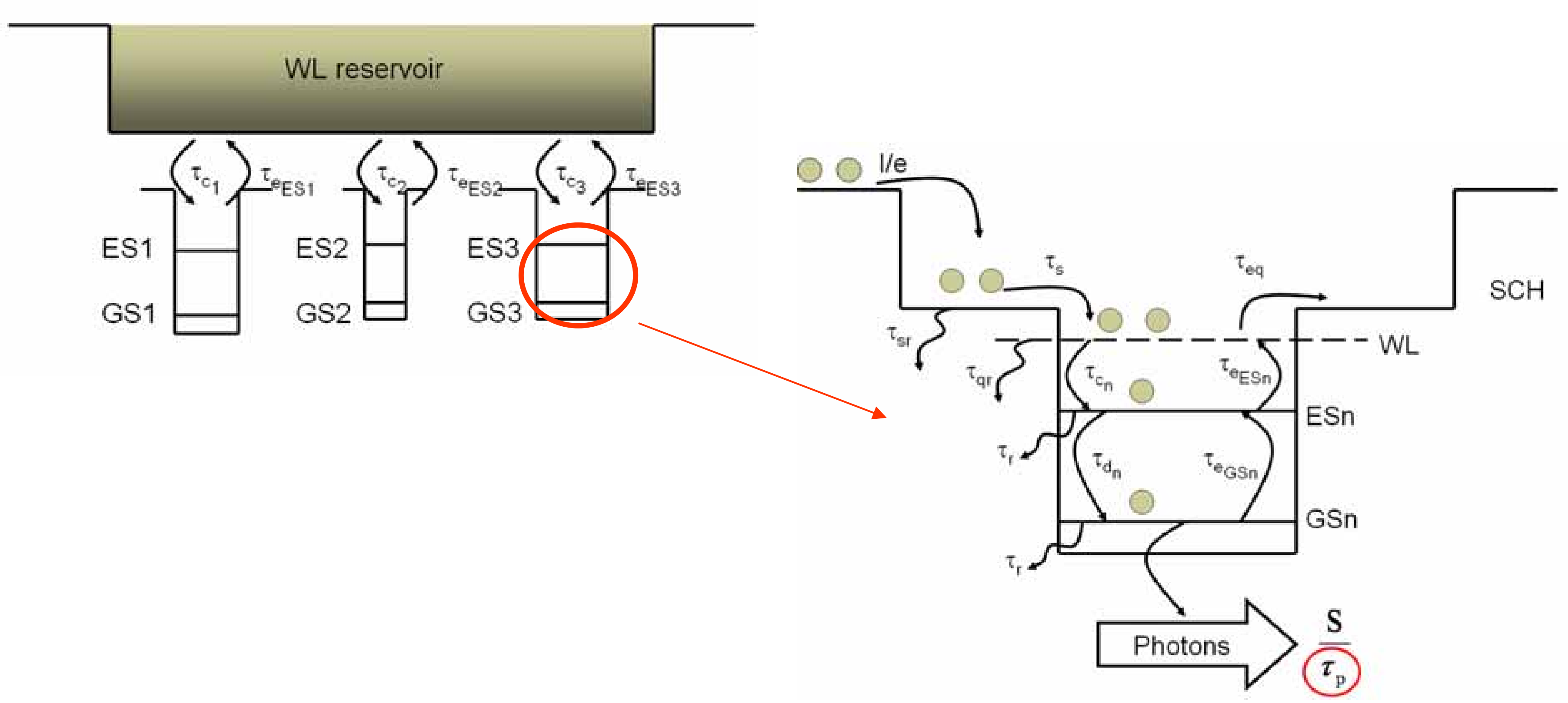
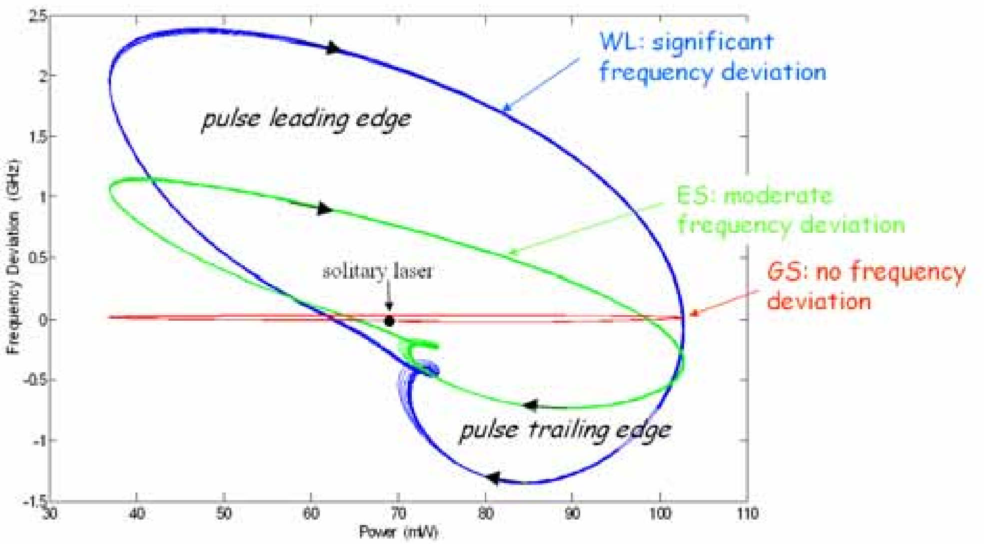
Carrier dynamics in QD laser

- *QD of different size are coupled together via the common WL*
- *Carrier in QDs are captured from the WL in the ES and relax down in the GS*
- *Lasing takes place only from GS (SLM laser)*



Conclusions from example #1

- The pulses generated by the instability experience more frequency variation during the pulse trailing edge respect to the pulse leading edge



- This is caused by the delay with the ES and WL carriers respond to decreasing power
- The frequency deviation respect to the solitary laser and the delay are more pronounced for the WL than the ES
- The GS can not cause instability because the frequency deviation respect to the solitary laser is negligible

Comparison with Qwell or bulk laser (1)

The “equivalent Qwell or bulk laser”:

- is modeled with one rate equation for carriers and one for the electric field

- is defined as <u>the solitary laser</u> with the same output power, relaxation oscillation frequency, damping factor and α-parameter of the QD laser

- these parameters are extracted from the analysis of a small perturbation of the solitary QD laser at the operation point

The results of QD laser simulations are compared with:

- the stability analysis results obtained by *J. Mørk, B. Tromborg, J. Mark, IEEE JQE, vol. 28, no. 1, January 1992*

- the simple analytic expression for stable operation condition by

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applied to the “equivalent QW laser”