Domain Decomposition Method for Electromagnetic Scattering Problems: Application to EUV Lithography

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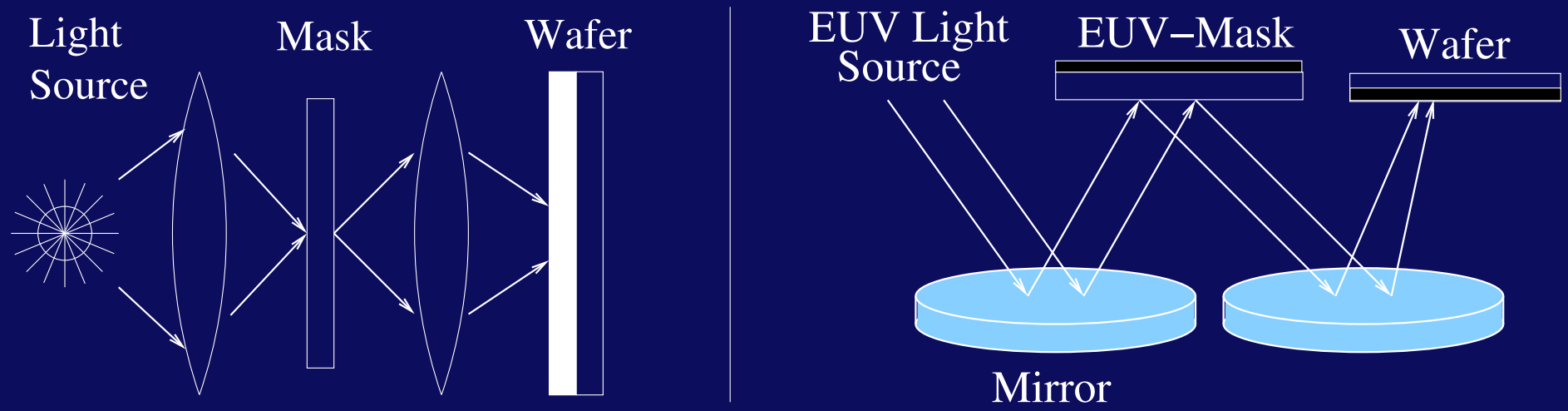
Matheon

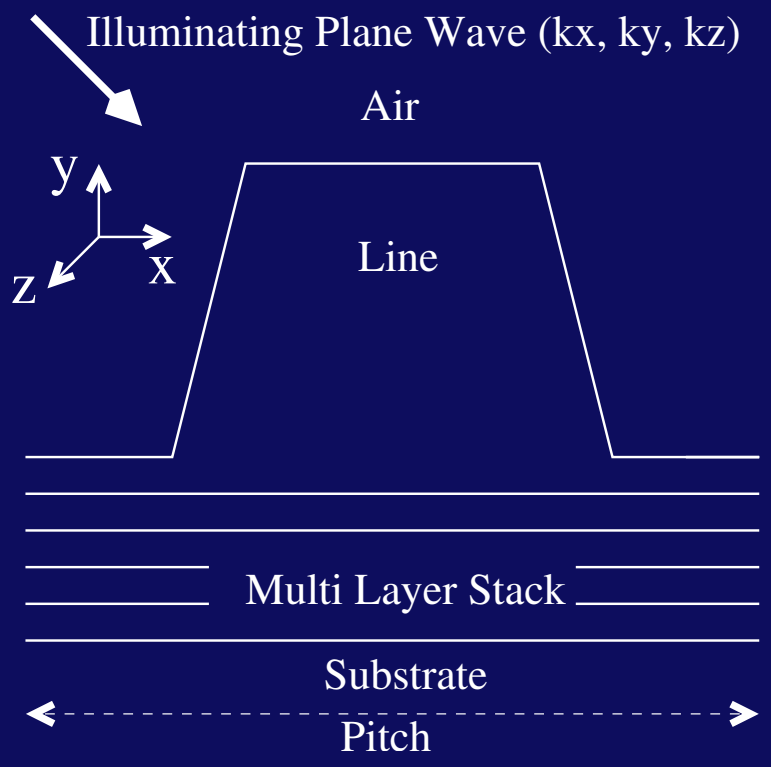
Photolithography

- Imprint the integrated circuits to a semi-conductor chip
- Transfer mask pattern by optical projection



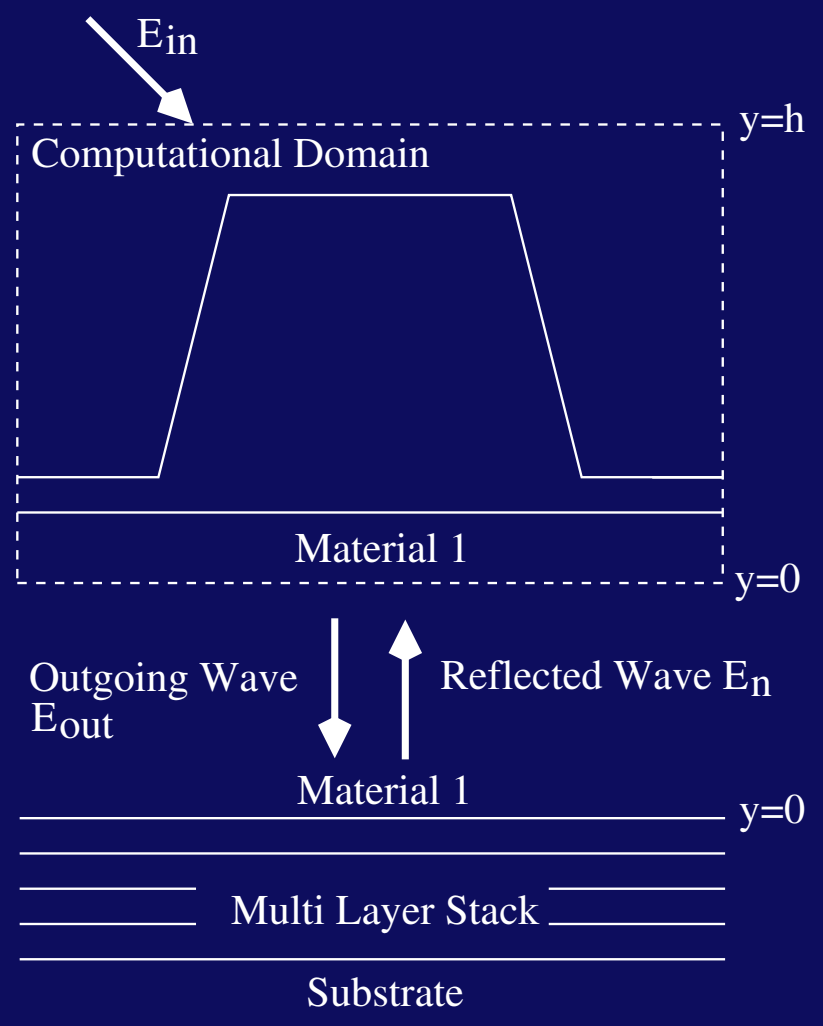
- Left: State-of-the-art tools, wavelengths of 250 and 190 nm. Minimum feature sizes down to 65nm.
- Right: Extreme Ultraviolet (EUV) radiation system, 13nm wavelengths. Dramatically reduced resist feature sizes.

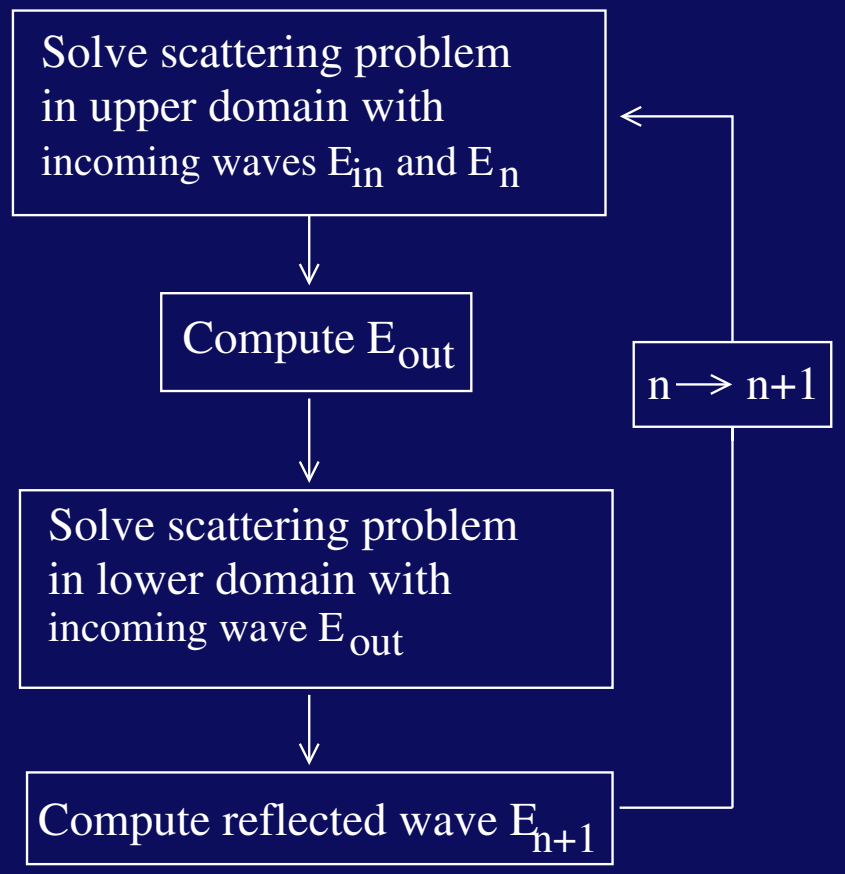
EUV Mask



- Multilayer coating serves as a mirror.
- Leads to huge computational domains.

Domain Decomposition





See also: Despres ('97), Toselli ('98), Colino & Joly ('00), Gander ('02)

DD: Building Blocks

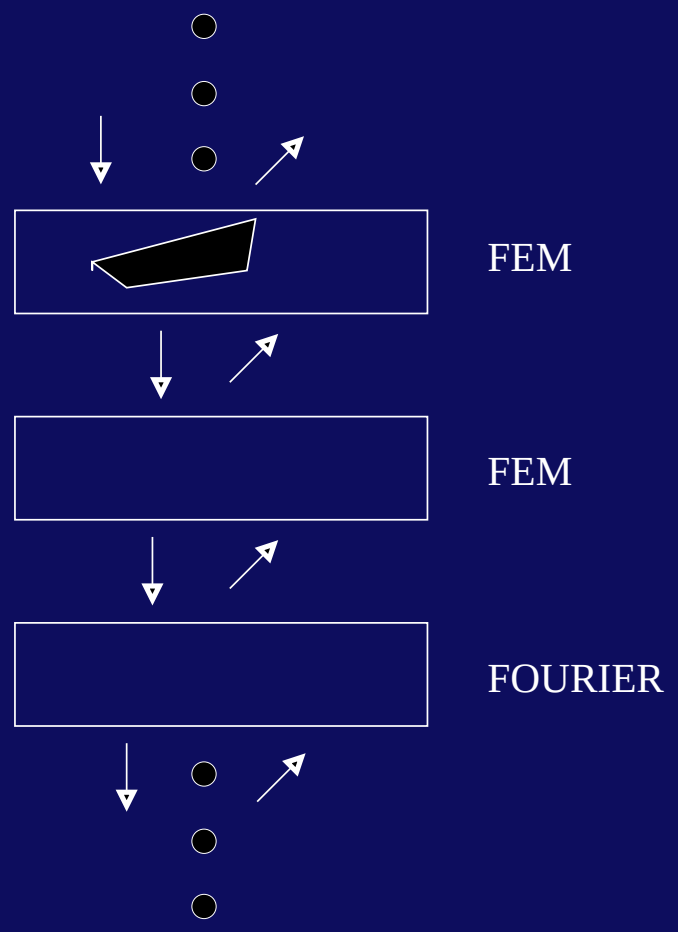
Scattering Problems:

- Finite elements with transparent boundary condition in structure blocks and for *perturbed* layer blocks.
- Fourier-mode expansion in *non-perturbed* layer block (semi-analytic solution)

Coupling Problems:

- FEM – FEM Coupling (with transparent boundary conditions!)
- FEM – Fourier Mode Coupling
- Fourier Mode – FEM Coupling

DD: Building Blocks



DD: Maxwell's Equations

Periodic and z–invariant Media:

$$\varepsilon(x+a,y) = \varepsilon(x,y), \quad \mu(x+a,y) = \mu(x,y)$$

Bloch periodic incoming field:

$$\mathbf{E}_{\text{inc}}(x+a,y,z) = \mathbf{E}_{\text{inc}}(x,y)e^{ik_x a}e^{ik_z z}$$

Maxwell's equation:

$$\mathbf{curl}_{k_z}\mu^{-1}\,\mathbf{curl}_{k_z}\mathbf{E} - \omega^2\epsilon\mathbf{E} = 0$$

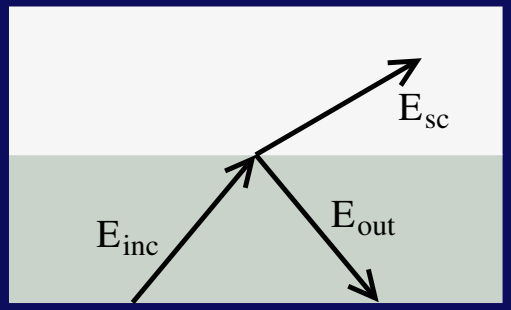
with $\mathbf{curl}_{k_z} = \mathbf{curl}\,, \partial_z \leftrightarrow ik_z$

Bloch Periodicity:

$$\mathbf{E}(x+a,y) = \mathbf{E}(x,y)e^{ik_x a}$$

Fourier Modes Expansion

Most simple situation: μ and ε are constant



Fourier expansion of $\mathbf{E}_{\mathsf{sc}}$:

$$\mathbf{E}_{\mathsf{sc}}(x,y) = \sum_n e^{i(n2\pi/a+k_x)x}\left(\vec{e}_{n,+}e^{ik_{y,n}y} + \vec{e}_{n,-}e^{-ik_{y,n}y}\right)$$

with $k_{y,n} = \sqrt{k_0^2 - (n2\pi/a + k_x)^2 - k_z^2}$

Radiation boundary condition: $\vec{e}_{n,-} = 0$

Anomalous Modes

- Real(k_y) $>$ (Radiating mode, Sommerfeld like boundary condition)

- Imag(k_y) $>$ (Evanescent mode)

- $k_y = 0$ (Anomalous mode)

Fourier Coupling

Fourier expansion of incoming field and reflected field:

$$\begin{aligned}\mathbf{E}_{\text{inc}}(x,y) &= \sum_n e^{i(n2\pi/a+k_x)x}\vec{e}_{n,\text{inc}}e^{ik_{y,n}y}\\ \mathbf{E}_{\text{out}}(x,y) &= \sum_n e^{i(n2\pi/a+k_x)x}\vec{e}_{n,\text{out}}e^{-ik_{y,n}y}\end{aligned}$$

- $|k_{y,n}| \geq threshold$: Dirichlet Coupling

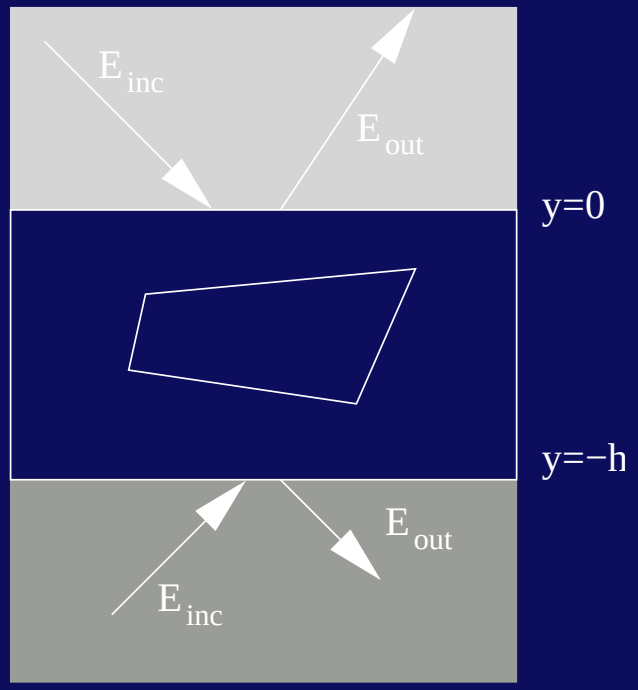
  $$\vec{e}_{n,\text{inc}} \times \vec{n} \mapsto (\vec{e}_{n,\text{out}} \times \vec{n}, ik_{y,n} \times \vec{e}_{n,\text{out}} \times \vec{n})$$

  No evaluation of $\mathbf{curl}\,\mathbf{E}_{\text{inc}} \times \vec{n}$.

- $|k_{y,n}| < threshold$: Mixed Dirichlet–Neumann Coupling
  Splitting into two polarization directions.
  Use weak evaluation of $\mathbf{curl}\,\mathbf{E}_{\text{inc}} \times \vec{n}$.

Finite Element Problem

$$\begin{aligned}\Omega_+ &= [0,a]\times[0,\infty)\\ \Omega &= [0,a]\times[-h,0]\\ \Omega_- &= [0,a]\times(-\infty,-h]\end{aligned}$$



Split field:

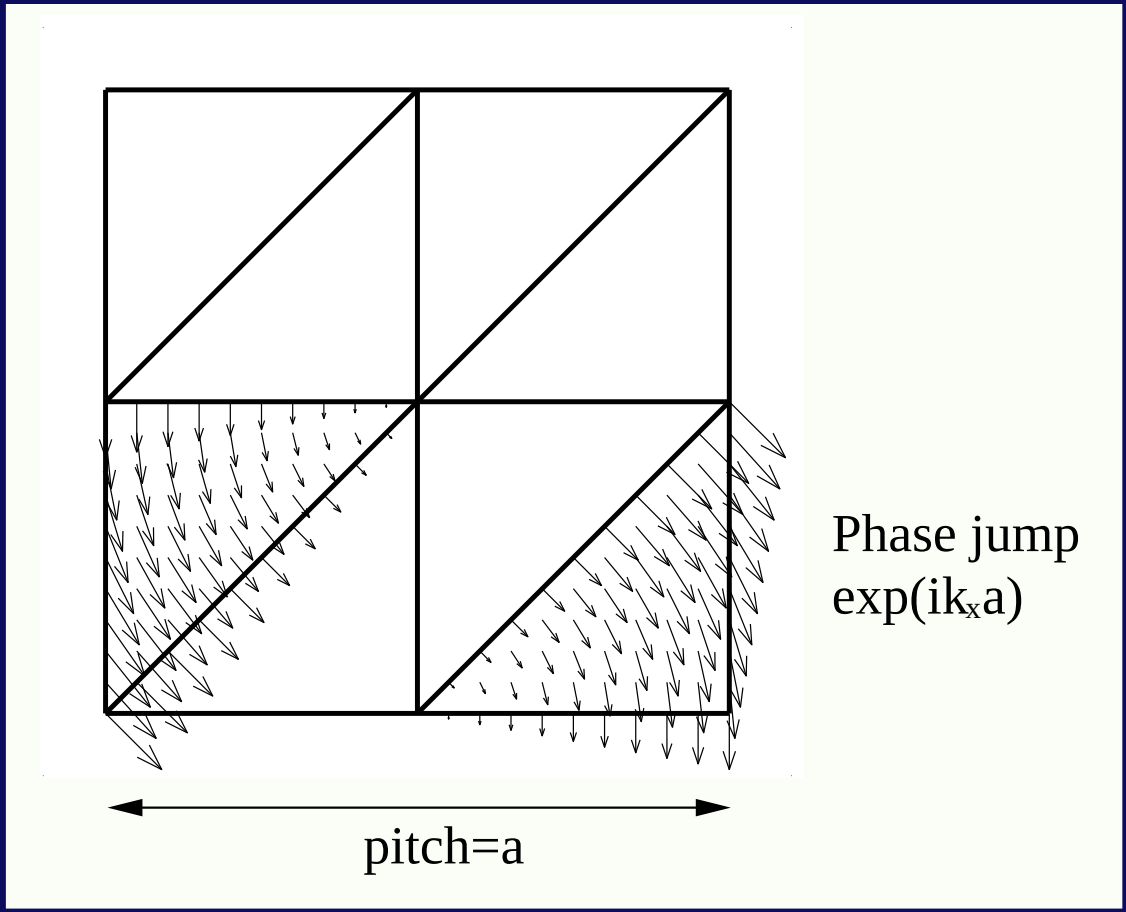
$$\mathbf{E} = \begin{pmatrix} \mathbf{E}_\perp \\ E_z \end{pmatrix}$$

with

$$\begin{aligned}\mathbf{E}_\perp &\in H_{\mathsf{B}}(\,\mathbf{curl}\,) := \left\{\mathbf{u}\in H(\,\mathbf{curl}\,)\,|\,\mathbf{u}(x+a,y) = e^{ik_x a}\mathbf{u}(x,y)\right\}\\ \mathbf{E}_z &\in H^1_{\mathsf{B}} := \left\{u\in H^1\,|\,u(x+a,y) = e^{ik_x a}u(x,y)\right\}\end{aligned}$$

See also: Elschner et al. (1999) Theory & (E_z, H_z)–formulation

Bloch Periodic Finite Elements



Discretization of $\mathbf{H}_\perp$ with Nedelec elements,
H_z with Lagrange elements.

Coupled Exterior–Interior System

$\Omega_+ = [0, a] \times [0, \infty)$

Interior Problem

$$\mathbf{curl}_{k_z} \mu^{-1} \mathbf{curl}_{k_z} \mathbf{E} - \omega^2 \varepsilon \mathbf{E} = 0$$

Exterior Problem

$$\mathbf{curl}_{k_z} \mu_+^{-1} \mathbf{curl}_{k_z} \mathbf{E}_{\mathrm{sc}} - \omega^2 \varepsilon_+ \mathbf{E}_{\mathrm{sc}} = 0$$

Matching condition at $y = 0$

$$\begin{aligned} \mathbf{E} \times \vec{n} &= (\mathbf{E}_{\mathbf{sc}} + \mathbf{E}_{\mathbf{inc}}) \times \vec{n} \\ \mu^{-1} \mathbf{curl}\, \mathbf{E} &= \left(\mu_+^{-1} \mathbf{E}_{\mathbf{sc}} + \mu_+^{-1} \mathbf{E}_{\mathbf{inc}}\right) \times \vec{n} \end{aligned}$$

Perfectly Matched Layer

Idea: Complex continuation ($y \to (1 + i\sigma)y$) of $\mathbf{E}_{\mathsf{SC}}(x, y)$

$$\mathbf{E}_{\mathsf{PML}}(x, y) = \sum_n \vec{e}_{n,+} e^{ik_{y,n}(1+i\sigma)y} e^{i(n2\pi/a+k_x)x}$$

Exponential decaying field:

$$|\mathbf{E}_{\mathsf{PML}}(x, y)| \sim e^{-\kappa y}$$

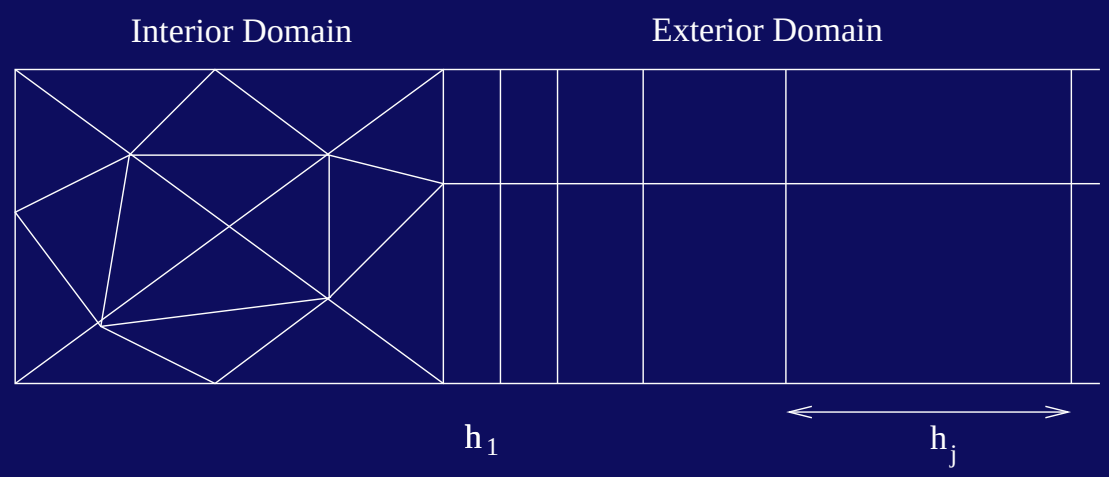
with $\kappa = \min_n\{\Re k_{y,n}, \Im k_{y,n}\}$.

Truncate exterior domain: $\Omega_\rho = [0, a] \times [0, \rho]$

$\kappa = 0$ in the presence of anomalous modes!

PML for anomalous modes

The (near) anomalous modes are slowly varying in y.
Idea: Use very, very large PML layer but very few discretization points.



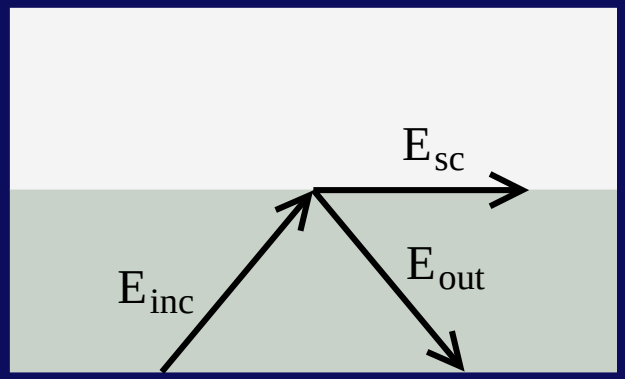
Heuristical choice of h_{j+1}: Determine $k_{y,\text{max}}$ such that

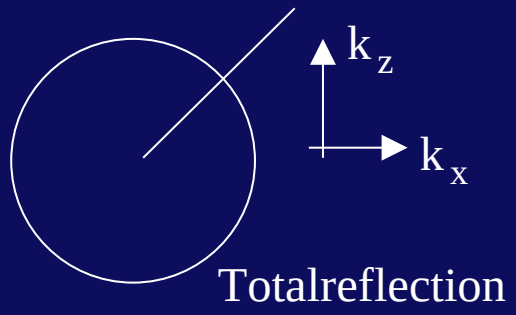
$$e^{-ik_{y,\text{max}}\rho_j} \leq threshold, \quad \rho_j = \sum_{j' \leq j} h_{j'}$$

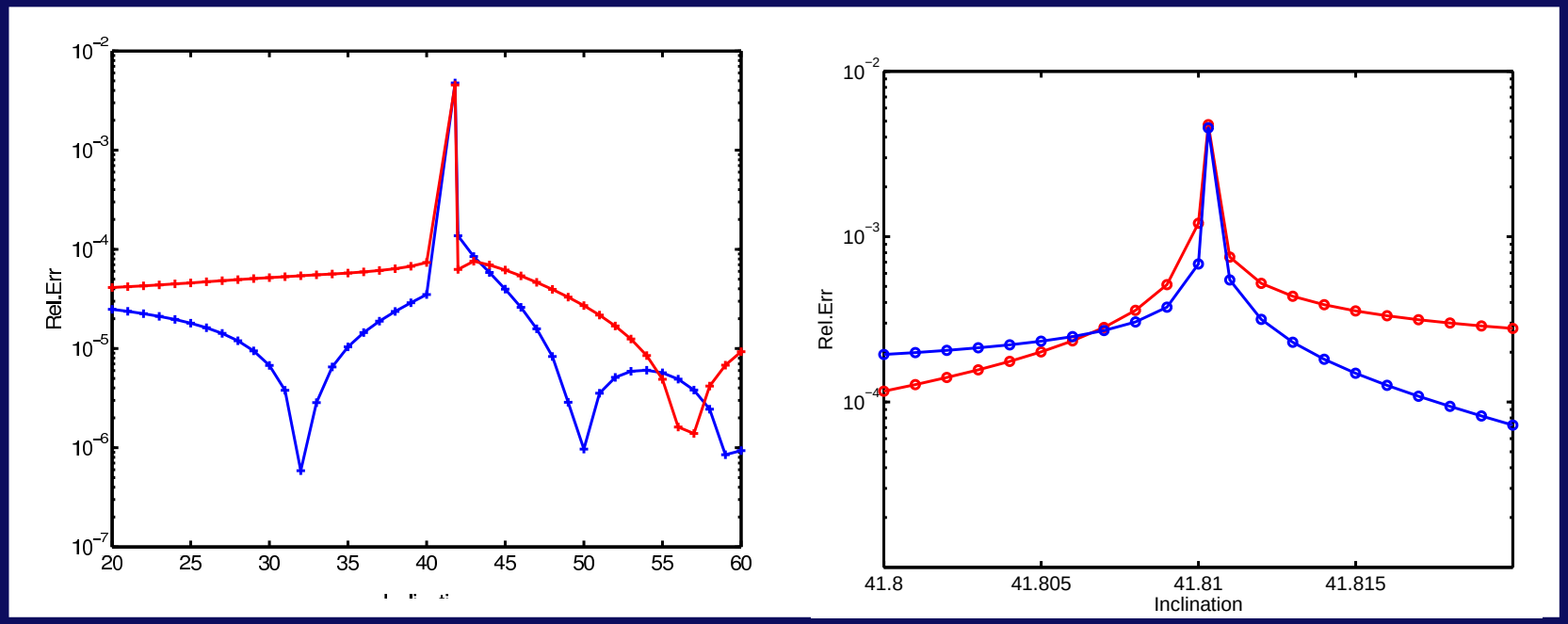
Waves with $\lambda_{\text{min}} <= 2\pi/k_{y,\text{max}}$ are sufficiently damped

$$\rightarrow h_j = \max\{h_1, \lambda_{\text{min}}/N\}$$

Total reflection

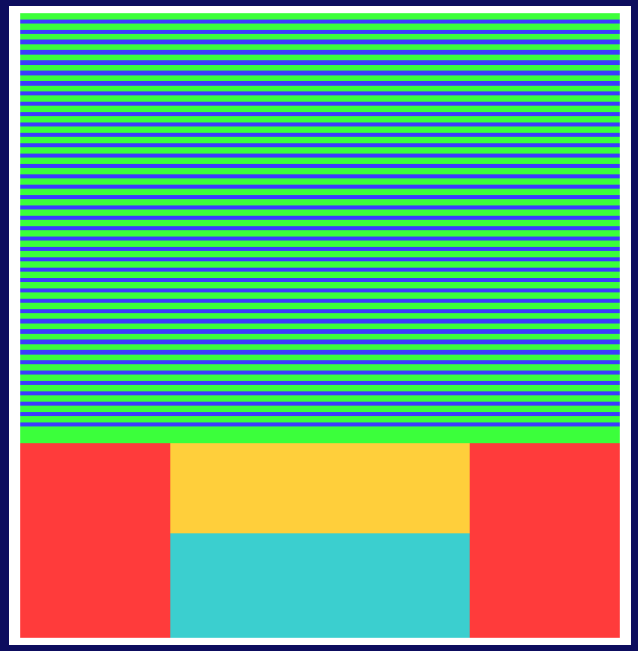


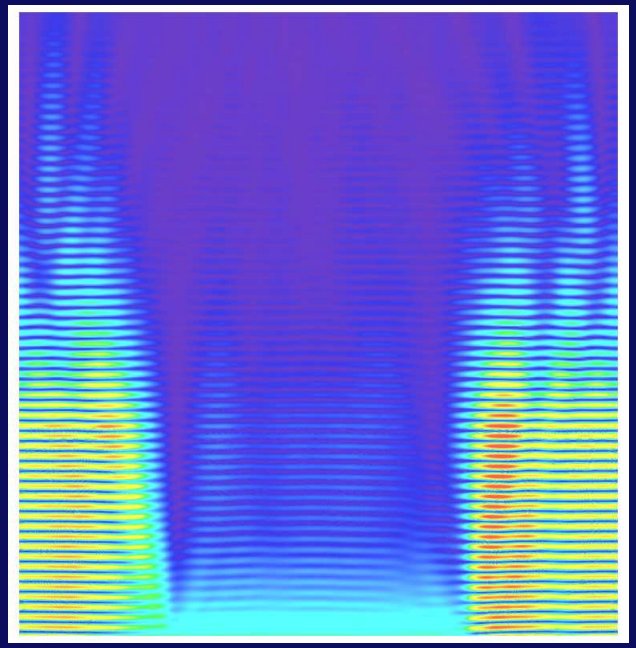


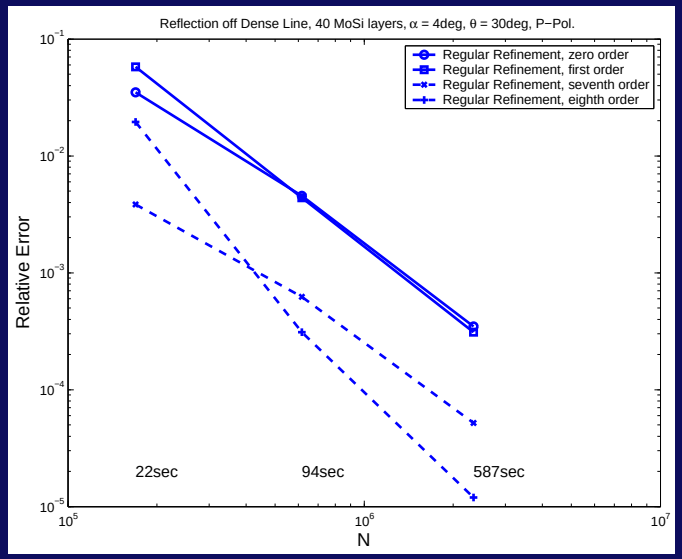


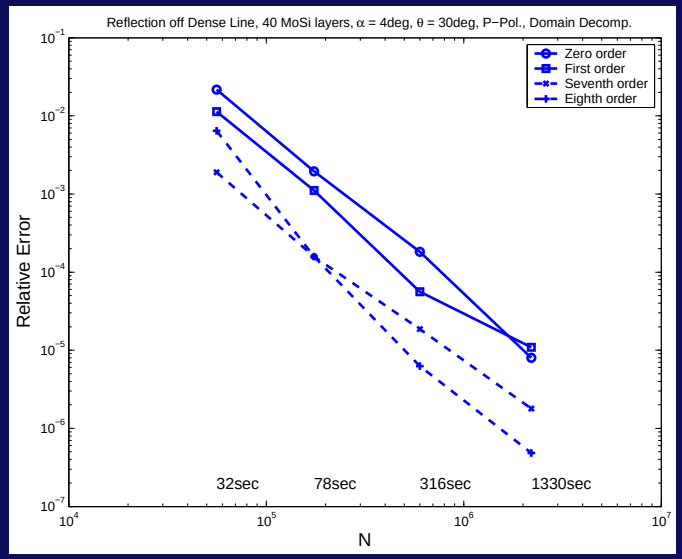
Error at wood anomaly: 4.78e-3, 1.1e-3, 2.9e-4, 7.2e-5
(uniform *interior* grid refinement)

Real world example









Outlook

- 3D Photolithography mask. All building blocks for domain decomposition available in JCMharmony (by JCMwave)

- Isolated Structures, e.g. contact holes. Finite Element solver available. Coupling of domains requires exterior field evaluation (pole condition).

Talk by Frank Schmidt et al.:

Benchmark of FEM, waveguide, and FDTD algorithms for rigorous mask simulation, Photomask Technology 2005, Monterey, 4 October 2005